Occupational Choice, Human Capital, and Financial Constraints*

Rui Castro† Pavel Ševčík‡

February 2024

Abstract

We study the aggregate productivity effects of firm-level financial frictions. Credit constraints affect not only production decisions, but also household-level schooling decisions. In turn, entrepreneurial schooling decisions impact firm-level productivities, whose cross-sectional distribution becomes endogenous. In anticipation of future constraints, entrepreneurs under-invest in schooling early in life. Frictions lower aggregate productivity because talent is misallocated across occupations, and capital misallocated across firms. Firm-level productivities are also lower due to schooling distortions. These effects combined account for between 36% and 68% of the U.S.-India aggregate productivity difference. Schooling distortions are the major source of aggregate productivity differences.

Keywords: Aggregate Productivity, Financial Frictions, Entrepreneurship, Human Capital, Misallocation.

JEL Codes: E24, I25, J24, O11, O15, O16, 047.

*We thank the editor and two referees for useful comments. We also thank our discussants Pedro Amaral, Irineu de Carvalho Filho, and Diego Restuccia for extremely helpful comments, as well as seminar attendants at the 2013 CEA, 2013 SED, 2013 Lubramacro, 2013 CMSG, 2014 Macroeconomics and Business CYCLE conference, 2015 Econometric Society World Congress, 2015 Midwest Macro Conference, 20th Economics Day at Ensai, 2016 Macroeconomics of Entrepreneurship conference in Montréal, Banco de Chile, Bank of Canada, Kentucky, Manitoba, McGill, Pittsburgh, PUC Chile, Toronto, VCU, Waterloo, and Western Ontario. Thanks to Francisco Adame for excellent research assistance. All errors are ours. Both authors acknowledge financial support from SSHRC. Castro acknowledges financial support from the Fonds Marcel-Faribault/Université de Montréal, and Ševčík from FRQSC.

†Corresponding author. Department of Economics and CIREQ, McGill University. Email: rui.castro@mcgill.ca. Web: rui-castro.research.mcgill.ca.

‡Département des sciences économiques, ESG UQAM. Email: sevcik.pavel@uqam.ca. Web: www.sevcik.uqam.ca.

1 Introduction

Total Factor Productivity (TFP) is the single most important factor accounting for the large cross-country income differences we see in the data (Klenow and Rodríguez-Clare, 1997; Hall and Jones, 1999; Caselli, 2005; Hsieh and Klenow, 2010). We evaluate the quantitative significance of financial frictions as a source of TFP differences. Entrepreneurs face a standard collateral constraint when raising business capital. Our main contribution is to consider also the role of entrepreneurial schooling decisions, and how they interact with financial frictions.

We view entrepreneurial human capital as a main determinant of firm-level productivity. Consistent with the evidence we provide, skills vary widely among entrepreneurs, with the more educated ones being better managers, more aware and skilled at implementing better management practices (Bloom and Reenen, 2007), and therefore operate more productive businesses. In this setting, future entrepreneurs under-invest in schooling in anticipation of credit constraints. They do so because investing in schooling is not very productive in small-sized firms, and also because the opportunity cost of schooling investments is high when resources could be used instead to build up collateral. In other words, entrepreneurs don't invest much in education since they realize they will be running a small family business; they prefer instead to work hard in order to save more. Further, schooling investments get misallocated. That is, those entrepreneurs with the best productivity potential are the ones who feel compelled to reduce schooling investments the most. We find that these two effects, schooling under-investment and schooling misallocation, play a very important quantitative role in accounting for the U.S.-India TFP difference. They jointly contribute to most of the model-generated variation.

Our model bridges two literatures/frameworks. One is a model of entrepreneurship with credit constraints, along the lines of Buera and Shin (2013) and Midrigan and Xu (2014), among others.[1] The other is a model of human capital accumulation along the lines of Erosa et al. (2010) and Manuelli and Seshadri (2014).

Like in the existing literature on entrepreneurship with credit constraints, finan-

[1] Other references include Castro et al. (2004, 2009), Jeong and Townsend (2007), Erosa and Hidalgo-Cabrillana (2008), Amaral and Quintin (2010), Buera et al. (2011), Greenwood et al. (2013), Moll (2014), Cole et al. (2016), and Moll et al. (2017).

cial frictions generate misallocation of talent across occupations. Poor individuals talented at entrepreneurship choose to become workers, since their firms would operate at an inefficiently small scale. Other individuals, not so talented at managing and operating a production technology, find it advantageous to do so if sufficiently wealthy. Further, capital gets misallocated across those individuals that do decide to become entrepreneurs. This is because with credit constraints firm size depends on entrepreneurial wealth, not just firm-level productivity. On top of these well-understood effects of credit constraints, our framework generates additional ones, stemming from adjustments in entrepreneurial schooling choices and in the distribution of firm-level productivities. A key feature of our setup is precisely that the distribution of firm-level productivities becomes endogenous, determined by entrepreneurial-level schooling decisions.

We quantify the role of these different effects of credit frictions on TFP. In line with the previous literature, we first calibrate our model to the U.S. and consider a scenario where the only fundamental difference between the U.S. and India is the overall degree of financial frictions. In this case, our model accounts for 36% of the U.S.-India TFP difference. A second calibration also lets the average productivity of the human capital accumulation technology vary across the U.S. and India in order to match the average years of schooling difference across these two countries. This results in a significant amplification of the effect of frictions, namely entrepreneurial schooling under-investment, and the model accounts for 68% of the observed TFP difference.

Our modelling of schooling decisions follows Erosa et al. (2010) and Manuelli and Seshadri (2014). These papers emphasize the role of cross-country TFP differences in generating variation in human capital outcomes (Manuelli and Seshadri, 2014, also consider cross-country variation in relative prices of capital and demographics). Our model shares the feature that, in addition to time, expenditure in goods (or resource-based education quality) is also a key input into the human capital accumulation process. As in these papers, the education quality margin in our model leads workers to invest less in education in countries with lower wages (due to tighter credit frictions in our case). In our paper credit frictions also discourage schooling investments among entrepreneurs, by reducing the marginal return to those investments. The latter mechanism is independent from the presence of an education quality margin in the human capital accumulation process.[2] More generally, rather than studying the implications of a given exogenous degree of cross-country TFP differences for schooling outcomes, our key contribution is to highlight the role of entrepreneurial schooling decisions in shaping TFP, when entrepreneurs are subject to financial frictions.

Bhattacharya et al. (2013) also consider entrepreneurial investment in managerial skills, in a setting with exogenously given distortions in firm size. Larger distortions discourage skill investments by managers. As in their paper, the distribution of firm-level productivities in our model arises endogenously from entrepreneurial investments in human capital. In contrast to their framework, firm size distortions are endogenous here, and depend on the wealth distribution. In our model, constrained entrepreneurs under-invest in schooling partially in order to self-finance. This mitigates physical capital misallocation across firms, a mechanism also emphasized by Midrigan and Xu (2014).

Mestieri et al. (2017) is the closest to our paper. They emphasize the same overall mechanism as we do: credit market imperfections affect entrepreneurial investments in both the firm and in human capital, in a way that can help understand production outcomes and household heterogeneity in developing countries. They provide evidence for Mexico consistent with our shared view: family wealth does matter for both kinds of investment, with poorer entrepreneurs running smaller businesses, and sacrificing their offspring education when operating a "modern sector" firm. They build a quantitative model similar to ours in spirit, featuring dynastic overlapping generations. The main difference between our two papers is in the focus. Their goal is to assess, in a model calibrated to Mexico, the life-cycle dynamics of entrepreneurship (increasing rates of entrepreneurship and increasing modern firm size over an individual's life cycle) and the sources of household

[2] An extensive literature deals with educational decisions under credit constraints. An early example is Galor and Zeira (1993), and more recent developments are in Lochner and Monge-Naranjo (2011) and Córdoba and Ripoll (2013). As in these papers, our credit constraints also act as a direct mechanism lowering education, namely among poorer individuals. The central role of credit constraints in our model, however, is in affecting entrepreneurial, not worker, schooling decisions. In other words, rather than increasing current schooling costs for poor workers, credit constraints in our model act primarily on lowering the net future benefit of schooling for poor entrepreneurs.

inequality (human capital investment under borrowing constraints generates high levels of income inequality and intergenerational persistence). Our focus is instead to assess the cross-country total factor productivity implications, in a model calibrated to the U.S. and India.

Finally, our paper is also related to the resource misallocation literature, namely Restuccia and Rogerson (2008), Hsieh and Klenow (2009), and Bartelsman et al. (2013). These authors examine the aggregate productivity consequences of misallocation generated by firm-specific taxes and subsidies. These taxes and subsidies are effectively stand-in, generic distortions, meant to capture deeper allocative problems. Our model concentrates on one such allocative problem: malfunctioning credit markets. We provide an explicit mapping between fundamental distortions coming out of our model, which have a structural interpretation, and the stand-in taxes and subsidies that are typically considered in this literature. In the process, we extend Hsieh and Klenow's (2009) framework for measuring the extent of resource misallocation. In our case, in addition to distortions to cross-firm input allocation, there are also distortions impacting physical productivity relative to the frictionless benchmark. The latter are induced by talent misallocation and by distortions to entrepreneurial schooling investments, and play the largest quantitative role.

The paper is organized as follows. Section 2 describes the model. Section 3 derives the aggregate productivity effects of distortions. Section 4 presents evidence on the significance of schooling for entrepreneurship. Section 5 describes the calibration procedure. Section 6 presents the quantitative results, and Section 7 concludes. The Appendices contain detailed information about the formal definition of equilibrium, some analytical properties of the model, the mapping between model and data, and the numerical procedure.

2 Model

2.1 The Environment

Consider an economy with measure one of altruistic dynasties. We abstract from differences in country size. Each individual lives for 2 periods, childhood and adulthood. The household, composed of a child and an adult parent, is the decision unit (unitary household model). We call childhood the period when schooling and investment decisions are made, and adulthood the period when the individual's main economic activity is carried out. Households value stochastic aggregate household consumption streams according to

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u\left(c_t\right), \tag{1}$$

where $\beta \in (0,1)$ is the time and generation discount factor. The period utility function u is of class C^2, strictly increasing, strictly concave, and satisfies the usual Inada condition.

In anticipation of our recursive formulation, we use primes to denote variables pertaining to the next generation, whereas those without primes refer to the current one. The household starts the period with wealth ω, and a draw of the child's abilities, current learning ability $z > 0$, and future entrepreneurial ability $x > 0$. The inter-generational ability transmission is governed by a first-order Markov chain with transition probabilities $\pi\left(z', x' | z, x\right)$.

Given the current state (ω, z, x), the household makes four decisions. First, it decides today's investment in the child's education, by choosing schooling time s and schooling expenditures e to produce human capital according to

$$h = A_h z \left(s^{\eta} e^{1-\eta}\right)^{\xi}, \tag{2}$$

with $\eta \in [0,1]$ and $\xi \in [0,1)$. A_h captures the aggregate efficiency of the schooling sector, which we set to 1 in benchmark case.

We follow Erosa et al. (2010) and Manuelli and Seshadri (2014) in considering expenditures as an input to human capital accumulation in addition to student time. This allows a worker's schooling time to increase with wages. With the presence of expenditures, higher wages increase the marginal gain from schooling investments by more than the marginal cost, since the price of the goods input is invariant to the wage.[3]

[3] As in Erosa et al. (2010), this also relies on the presence of tuition costs, which we also model.

Second, the household decides today's saving for next period, by purchasing bonds in net amount q at unit price $1/(1+r)$. We assume ability shocks are uninsurable.[4] Third, it decides the child's occupation for next period, whether to become an entrepreneur or a worker. Workers supply their human capital at the going wage rate. Entrepreneurs manage their own firms and are the residual claimants of profits. Fourth, if the decision is to become an entrepreneur next period then the household also needs to raise capital, possibly with external funds, and hire labor in order to run the firm.

All production is carried out by entrepreneurs according to

$$y = xh^{1-\gamma}\left(k^{\alpha}l^{1-\alpha}\right)^{\gamma}, \tag{3}$$

with $\alpha, \gamma \in (0,1)$, where k and l denote the physical capital and labor inputs. It is convenient to define entrepreneurial, or firm-level productivity as $a^{1-\gamma} \equiv xh^{1-\gamma}$, where x is determined by luck and h is the entrepreneur's human capital level. Physical capital depreciates at rate $\delta \in (0,1)$.

2.2 Household's Problem

We focus on stationary equilibria, in which prices and the cross-sectional distribution over individual states are time-invariant. Denote by w the wage rate (unit price of human capital) and by r the real interest rate. We begin by formulating the household's problem conditional on the child's occupational choice. Notice that, since all uncertainty is resolved at the start of an individual's life, the occupational choice can be made right then.

Conditional on the child becoming a worker next period, the worker-household's

---

Tuition costs prevent the marginal gains and costs from an additional year of schooling to both vary proportionally with the current level of human capital, allowing schooling years to vary with both learning ability z and, via school quality adjustments, wages.

[4]Given our assumption on the resolution of uncertainty, saving is contingent upon the child's abilities, namely next period's entrepreneurial ability. We abstract from precautionary saving behavior associated with entrepreneurial ability risk in order to streamline the analysis. This allows us to characterize the household investment decisions via simple non-arbitrage conditions.

(worker, for short) problem can be written recursively as:

$$v^{w}(\omega, z, x) = \max_{c,e,s,q} \left\{ u(c) + \beta \sum_{z',x'} \pi(z', x'|z, x)\, v(\omega', z', x') \right\} \tag{Pw}$$

subject to (2) and

$$c + ws\bar{l} + e + \frac{1}{1+r}q = w\psi h(1-s) + \omega \tag{4}$$

$$s \leq \bar{s} \tag{5}$$

$$q \geq -\lambda\phi \max\{\omega, 0\} \tag{6}$$

$$\omega' \equiv wh + q. \tag{7}$$

Equation (4) is the budget constraint. The term $ws\bar{l} + e$ is the direct cost of investing in the child's education, tuition fees $ws\bar{l}$ ($\bar{l}$ is the total teacher input per unit of student time, a parameter) plus expenditures in education quality e. Teacher's effective time is not an input into human capital production, only student time is. Expenditures in goods capture direct costs such as books and computers. On the right-hand-side, $w\psi h(1-s)$ is the child's labor earnings, where $\psi \in (0,1)$ captures increasing labor earnings over an individual's lifetime due to experience.

Equation (5) is the child's time constraint. We impose an upper bound $\bar{s} \leq 1$ for quantitative purposes, since individuals do not normally spend their entire early life studying.

Households are subject to an inter-period *household credit constraint* given by (6). They can only contract debt up to a multiple $\lambda\phi \geq 0$ of their wealth.[5] When $\phi = 0$ no borrowing is allowed, and investment must be funded out of the household's wealth; when $\phi = \infty$ (provided $\lambda > 0$, which we assume) access to household credit is unconstrained. Equation (7) defines the initial wealth of the next household in the

[5]This constraint can be motivated by a simple static limited enforcement problem. Suppose a household decides whether to default on the loan repayment $-q$. The only penalty is that financial intermediaries may seize a fraction $\nu \in [0,1]$ of initial wealth $\omega > 0$, net of q. Intermediaries then require that the gain from defaulting does not exceed the cost, that is $-(1-\nu)q \leq \nu\omega$. This yields (6) with $\phi\lambda \equiv \nu/(1-\nu) \geq 0$. The main advantage from using this simple specification is tractability. It shares with self-enforcing limits based on dynamic incentives (Kehoe and Levine, 1993) the key feature that richer households are able to borrow more.

dynastic line, conditional on the fact that next period's parent will be a worker.

Similarly, conditional on the child becoming an entrepreneur next period, the entrepreneur-household's (entrepreneur, for short) problem reads:

$$v^{e}\left(\omega, z, x\right) = \max_{c,e,s,q} \left\{ u\left(c\right) + \beta \sum_{z',x'} \pi\left(z', x' | z, x\right) v\left(\omega', z', x'\right) \right\} \tag{Pe}$$

subject to (2)–(6) and a new definition of household's wealth based upon entrepreneurial profits

$$\omega' \equiv \Pi\left(q, h, x\right) + q, \tag{8}$$

where

$$\Pi\left(q, h, x\right) = \max_{k,l \geq 0} \left\{ a^{1-\gamma} \left(k^{\alpha} l^{1-\alpha}\right)^{\gamma} - \left(r + \delta\right) k - wl \right\} \tag{Pf}$$

subject to

$$k \leq \lambda \frac{q}{1+r}, \tag{9}$$

with $\lambda \geq 1$. Entrepreneurs hire capital and labor to maximize profits, subject to an intra-period *capital constraint*. The maximum level of capital an entrepreneur can employ in production is given by a multiple λ of the household's second period assets, which acts as collateral.[6] When $\lambda = 1$ no external funding is allowed, and capital is solely determined by internal funds. When $\lambda = \infty$ financial markets work perfectly, and capital is not constrained by wealth.

Financial frictions affect the model via (6) and (9). The parameter λ governs the overall extent of financial frictions in the economy, whereas ϕ controls the household credit constraint. We choose this formulation to reflect the possibility that seizing

[6]We assume future profits are not pledgable as collateral. Constraint (9) therefore implies that households that borrow today will not be able to run a firm tomorrow. As a result, only children from sufficiently wealthy backgrounds can aspire to become entrepreneurs. Similarly to (6), the constraint (9) may be motivated by a simple static limited enforcement problem. As in Buera and Shin (2013), suppose households borrow k from financial intermediaries against collateral $q/(1+r)$, and then have a decision whether to default. The only penalty is that intermediaries may seize the entire collateral, plus a fraction of κ of k. No default requires $(1-\kappa)k \leq q/(1+r)$, which yields (9) with $\lambda \equiv 1/(1-\kappa) \geq 1$. Related work using identical collateral constraints include Evans and Jovanovic (1989), Moll (2014) and Moll et al. (2017).

wealth upon default, for example, might be easier for one type of credit compared to the other. In our quantitative analysis we let λ vary across countries while fixing ϕ.

The household's occupational choice for the child next period is then

$$v(\omega, z, x) = \max\{v^w(\omega, z, x,), v^e(\omega, z, x)\}. \tag{10}$$

Appendix A defines the stationary recursive competitive equilibrium.

3 Aggregate Productivity Implications of Financial Frictions

Our first task is to obtain expressions illustrating how financial frictions affect investment decisions in our model. We show in particular how the capital constraint that entrepreneurs anticipate later in life while producing distorts schooling investments when young. We then use the expressions encapsulating such distortions to derive the aggregate TFP implications.

3.1 Production

Conditional on their human capital, entrepreneurs hire labor and capital to maximize profits. The presence of the capital constraint implies the profit function:

$$\Pi(q, h, x) = \begin{cases} \Pi^*(h, x) & \text{if } q \geq q^*(h, x) \quad \text{(unconstrained)} \\ \Pi^c(q, h, x) & \text{else} \quad \text{(constrained)}, \end{cases}$$

where $q^*(h, x)$ is a threshold level of assets beyond which the capital constraint does not bind. Appendix B provides the expressions for threshold assets, as well as for the constrained and unconstrained profit functions. The constrained profit function is increasing in accumulated assets since a higher q allows the entrepreneur to raise more capital and increase the scale of the firm closer to its optimal level.

3.2 Schooling/Saving Decisions

Individuals can invest either through bonds, or by spending time and resources on schooling. Our timing assumption allows us to characterize these different investment opportunities in terms of simple non-arbitrage equations that transpire from the first-order optimality conditions for problems (Pw) and (Pe) with respect to s, e, and q (Appendix C). Schooling time is an implicit function $s = s(e)$ of schooling expenditures,

$$w\left(\bar{l} + \psi h\right) = \frac{\eta}{1-\eta}\frac{e}{s},$$

where s is strictly increasing in e since both inputs are complements. Replacing in (2) yields human capital $h = h(e)$.

Our key non-arbitrage condition equates the returns to saving and to human capital accumulation:

$$(1-\eta)\,\xi\frac{h}{e}\omega_2'\left(q,h,x\right) = \left(1+r\right)p_e\omega_1'\left(q,h,x\right), \tag{11}$$

where ω_1' and ω_2' are the partial derivatives of future wealth with respect to the first and the second arguments, capturing the returns to saving and to human capital, respectively. For convenience we denote the shadow unit price of schooling expenditures by

$$p_e = p_e(e) \equiv 1 - w\psi\left(1-s\right)\left(1-\eta\right)\xi\frac{h}{e}, \tag{12}$$

which equals the unit of foregone consumption net of the marginal increase in first-period earnings. Specializing (11) for each occupation allows us to characterize the optimal schooling decisions for workers and entrepreneurs.

3.2.1 Worker-Household

For workers we have

$$\omega_1'\left(q,h,x\right) = 1 \text{ and } \omega_2'\left(q,h,x\right) = w. \tag{13}$$

Since wages are linear in worker's human capital, returns to human capital accumulation are constant. An interior optimum for schooling expenditures solves:

$$\frac{w}{1+r}(1-\eta)\,\xi\frac{h}{e}=p_e. \tag{14}$$

The left-hand-side is the discounted future benefit of investing an extra unit of the final good on education, which is the wage rate times the marginal increase in human capital. The right-hand-side is the marginal cost.

When the borrowing constraint binds ($q=-\lambda\phi\omega$), optimal expenditures cannot be pinned-down by (11), and are instead the solution to a dynamic optimization problem. Schooling investments are then a function not just of learning ability, but also of current wealth ω.

3.2.2 Entrepreneur-Household

The capital constraint (9), together with the condition that $k \geq 0$, implies that entrepreneurs will always have $q > 0$ and therefore the constraint on household credit will never bind. We have:

$$\omega_1'(q,h,x)=\begin{cases}1+\frac{\partial\Pi^c}{\partial q}(q,h,x) & \text{if constrained}\\ 1 & \text{if unconstrained,}\end{cases} \tag{15}$$

and

$$\omega_2'(q,h,x)=\begin{cases}B(q)\,\frac{1-\gamma}{1-\gamma(1-\alpha)}a^{\frac{1-\gamma}{1-\gamma(1-\alpha)}}h^{-1} & \text{if constrained,}\\ Ax^{\frac{1}{1-\gamma}} & \text{if unconstrained,}\end{cases} \tag{16}$$

where the expressions for $B(q)$ and A, which depend on parameters and equilibrium prices, are given in Appendix B. From here we can deduce how the marginal returns to physical and human capital accumulation vary with the entrepreneur's saving q.

Proposition 1. *Given h, capital-constrained entrepreneurs (with $q < q^*(x,h)$) face a higher marginal return to physical capital accumulation and a lower marginal return to human capital accumulation than unconstrained entrepreneurs (with $q \geq q^*(x,h)$).*

Proof. The first part follows from (15), the fact that $\partial \Pi^c (q, h, x) / \partial q$ is decreasing in q, and that $\partial \Pi^c (q^* (h, x), h, x) / \partial q = 0$. The second part follows from (16), the fact that $B(q)$ is increasing in q, and that

$$B (q^* (h,x)) \frac{1-\gamma}{1-\gamma(1-\alpha)} a^{\frac{1-\gamma}{1-\gamma(1-\alpha)}} h^{-1} = A x^{\frac{1}{1-\gamma}}.$$

□

The first part of Proposition 1 comes from the fact that, for capital-constrained entrepreneurs, saving relaxes the capital constraint and allows them to expand their firms closer to the optimal unconstrained scale. The second part holds because human and physical capital are complements in production. Capital-constrained entrepreneurs employ less physical capital, making human capital less productive.

Proposition 1 establishes the central mechanism in our paper, that the anticipation of the capital constraint distorts saving and schooling decisions of entrepreneurs early in life. Constrained households therefore have an incentive to save more and invest less in education compared to unconstrained ones.

Substituting ω_1' and ω_2' for unconstrained entrepreneurs into (11) yields their condition for optimal schooling expenditures:

$$\frac{A}{1+r} (1-\eta) \xi \frac{a}{e} = p_e.$$

This condition is analogous to (14), with the left-hand side representing now the discounted marginal increase in future profits from investing an additional unit of the final good on schooling.

For constrained entrepreneurs we obtain:

$$\frac{B(q)}{1+r} (1-\eta) \xi \frac{1-\gamma}{1-\gamma(1-\alpha)} \frac{a^{\frac{1-\gamma}{1-\gamma(1-\alpha)}}}{e} = p_e \left(1 + \frac{\partial \Pi^c}{\partial q} (q, h, x)\right).$$

Compared to the unconstrained case, the marginal gain from investing in education is lower, and decreasing returns set in faster (Proposition 1). The marginal cost is also higher, since investing in education sacrifices wealth accumulation, which lowers firm capital and hence profits. Optimal spending in education therefore depends on household wealth, via saving q. More wealth helps relax the capital constraint, and reduces investment and schooling distortions.

3.3 Input Misallocation and Firm–Level Productivity Effects

We now characterize the input misallocation and the firm–level productivity effects stemming from the capital constraint. We borrow from the existing literature on input misallocation, namely Hsieh and Klenow (2009), to map these effects into aggregate TFP.

Our strategy follows in two steps. First, we show that the generic production distortions considered by Hsieh and Klenow (2009) have a structural interpretation in terms of our model. Second, we generalize their framework in the sense that model-based TFP differences are decomposed into not just an input misallocation effect, but also firm–level productivity effects. The latter features the distortions introduced via entrepreneurial investments in human capital. We are ultimately able to obtain a decomposition of the aggregate total factor productivity (TFP) effects into easily interpretable components.

3.3.1 Basic Model Wedges

Focus on entrepreneurs. The capital constraint introduces what amounts to individual-level wedges on the optimal conditions for saving and human capital expenditures. We call these structural distortions *basic model wedges*. We then show how they map into *proxy*, or stand-in misallocation *wedges*. The latter look like the generic, non-structural wedges featured in much of the misallocation literature, for example Restuccia and Rogerson (2008), Hsieh and Klenow (2009) and Bartelsman et al. (2013), among many others.

We summarize the effect of the capital constraint on an entrepreneur's optimality conditions (Appendix C) via two basic individual-specific wedges, labeled τ_q and

τ_h. That is, we can rewrite these conditions simply as

$$u'(c) = \beta(1+r)\left(1+\tau_q\right)\sum_{z',x'}\pi(z',x'|z,x)\,v_1(\omega',z',x'), \tag{17}$$

$$p_e u'(c) = \beta(1-\eta)\,\xi\,(1-\tau_h)\,A\frac{a}{e}\sum_{z',x'}\pi(z',x'|z,x)\,v_1(\omega',z',x'). \tag{18}$$

As long as basic wedges subsume the distortions affecting the marginal value of saving and human capital, (15) and (16), then these simple optimality conditions deliver the solution to the original problem. The appropriate wedges, which are functions of the current individual state, are defined by comparing the optimality conditions for constrained and unconstrained entrepreneurs:

$$\tau_q = \begin{cases} \frac{\partial \Pi^c(q,h,x)}{\partial q} & \text{if constrained,} \\ 0 & \text{if unconstrained,} \end{cases}$$

$$\tau_h = \begin{cases} 1 - \frac{B(q)}{A}\frac{1-\gamma}{1-\gamma(1-\alpha)}a^{-\frac{\alpha\gamma}{1-\gamma(1-\alpha)}} & \text{if constrained.} \\ 0 & \text{if unconstrained,} \end{cases}$$

The wedge $\tau_q \geq 0$ acts like a subsidy to saving, capturing the fact that whenever the capital constraint binds, an increase in saving today relaxes it and increases profits tomorrow. The wedge $\tau_h \in [0,1]$ acts like a tax on the returns to schooling, capturing the fact that human capital is less productive for constrained entrepreneurs. They have lower physical capital, which is complementary to human capital.

3.3.2 Proxy Production Wedges

We now recast the firm's problem as in Hsieh and Klenow (2009). We call it the proxy firm problem:

$$\Pi = \max_{k,l\geq 0}\left\{(1-\tau_a)\,p\,(a^*)^{1-\gamma}\left(k^\alpha l^{1-\alpha}\right)^\gamma - (1+\tau_k)(r+\delta)\,k - wl\right\}, \tag{Pf$'$}$$

where we define potential productivity as $(a^*)^{1-\gamma} \equiv x\,(h^*)^{1-\gamma}$, with h^* being the human capital level that would emerge if the capital constraint did not bind, and p the output price, which may be normalized to 1 in our setup.

We label τ_a and τ_k the individual-level *proxy wedges*, in the sense that they stand in for the fundamental distortions affecting the economy. τ_a captures distortions along the potential revenue (i.e. based on potential productivity) vs cost margin, whereas τ_k captures distortions along the capital vs labor input cost margin. Our task is now to infer proxy wedges from basic wedges, through the unique mapping between the two. The proxy firm problem (Pf′) yields the same solution as the original one (Pf) when

$$1 - \tau_a = \left(\frac{h}{h^*}\right)^{1-\gamma}$$
$$1 + \tau_k = 1 + \frac{\zeta}{r + \delta},$$

where ζ is the multiplier on the capital constraint.

Applying the Envelope theorem and using the definition of τ_q:

$$1 + \tau_k = 1 + \frac{\tau_q (1 + r)}{\lambda (r + \delta)}.$$

Under certain parametric restrictions, τ_a is also an explicit function of basic wedges, which helps build intuition. This is the case when $\psi = 0$, so that $p_e = 1$. Further assuming that the time constraint is slack, optimal schooling time s is proportional to expenditures e. We then obtain

$$1 - \tau_a = \left(\frac{1 - \tau_h}{1 + \tau_q}\right)^{\frac{(1-\gamma)\xi}{1-\xi}}.$$

The last two expressions allow us to structurally interpret the two proxy wedges. First, $\tau_k \geq 0$ amounts to a tax on capital, since the capital constraint increases the shadow rental price of capital. Second, $\tau_a \in [0, 1]$ amounts to a reduction in a firm's physical output, since the capital constraint decreases actual firm-level productivity $a^{1-\gamma}$ below potential, by discouraging entrepreneurial schooling investments. The total disincentive to investing in human capital is captured by the composite distortion $(1 - \tau_h)/(1 + \tau_q)$. It amounts to a positive tax since (i) capital-constrained entrepreneurs run smaller firms, reducing the returns to investing in human capital, and (ii) for these households, accumulating wealth relaxes the capital constraint, and therefore commands a higher return compared to investing in human capital.[7]

We can get further insight when $\bar{l} = 0$ (and $\psi > 0$), assuming again a slack time constraint, in which case

$$1 - \tau_a = \left(\frac{1 - \tau_h}{1 + \tau_q}\right)^{\frac{1-(1-\gamma-\eta)\xi}{\xi(1-\gamma)^2}} \left(\frac{p_e^*}{p_e}\right)^{\frac{1-(1-\gamma-\eta)\xi}{\xi(1-\gamma)^2}}, \tag{19}$$

where p_e^* is defined as the shadow unit price of schooling expenditures ignoring credit constraints. Although a closed form is not available (p_e is itself a function of basic distortions), this formulation helps illustrate the role of A_h in amplifying the effect of basic distortions on the composite τ_a. A lower A_h decreases the entrepreneur's ability to generate first-period earnings out of a given investment in human capital, making schooling more expensive, see (12). Both p_e and p_e^* therefore increase, with the effect on p_e being stronger since entrepreneurs spend more time working early in life when credit frictions are active. τ_a therefore increases for given basic wedges. In short, when the environment makes it difficult for entrepreneurs to self-finance when young, our basic mechanism is amplified: they cannot build sufficient collateral, which hurts schooling. It follows from this same reasoning that entrepreneurs with high learning ability z (and higher potential productivity) also face larger distortions, generating a misallocation of schooling investments which will negatively impact aggregate productivity.

The production technology underlying the stand-in problem (Pf′) is $y \equiv (1 - \tau_a)(a^*)^{1-\gamma}\left(k^\alpha l^{1-\alpha}\right)^\gamma$. We follow Foster et al. (2008) and Hsieh and Klenow (2009) in defining a firm's (actual) physical productivity TFPQ and revenue

[7] Why do financial frictions generate a disincentive to human capital accumulation ($\tau_a > 0$)? Why don't frictions encourage entrepreneurs to invest more in schooling, and therefore generate higher first-period labor earnings per working time? Both are in fact feasible options for entrepreneurs in our model to generate higher savings and more self-financing. However, frictions increase the shadow interest rate ($\tau_q > 0$), and the non-arbitrage condition (11) needs to hold. Since investment in schooling exhibits decreasing marginal returns, this can only be the case if schooling declines.

productivity TFPR as[8]

$$\text{TFPQ} \equiv \frac{y}{\left(k^{\alpha} l^{1-\alpha}\right)^{\gamma}} = (1-\tau_a)\,(a^*)^{1-\gamma}$$
$$\text{TFPR} \equiv \frac{py}{k^{\alpha} l^{1-\alpha}}.$$

TFPR captures firm-specific deviations from marginal product equalization, and therefore the extent of capital misallocation.

From the optimality conditions

$$\gamma(1-\alpha)\left(\frac{k}{l}\right)^{\alpha} \text{TFPR} = w$$
$$\gamma\alpha\left(\frac{k}{l}\right)^{\alpha-1} \text{TFPR} = (1+\tau_k)\,(r+\delta)\,,$$

we obtain that revenue productivity

$$\text{TFPR} \propto (1+\tau_k)^{\alpha}\,.$$

Absent frictions, $\tau_q = \tau_h = 0$ and $p_e = p_e^*$ for every individual. Therefore $\tau_a = \tau_k = 0$. In this case the distribution of TFPR is degenerate, and the distribution of TFPQ reflects only individual heterogeneity in abilities among households selecting into entrepreneurship. With frictions, the distribution of TFPR becomes dispersed, reflecting physical capital misallocation, and the distribution of TFPQ shifts to the left, reflecting lower levels of entrepreneurial human capital for constrained entrepreneurs. These features become more pronounced with a tighter capital constraint.

Figure 1 plots the distributions of TFPR and TFPQ in our model, for both the U.S. and the India (benchmark) calibrations. A tighter capital constraint in India generates significant misallocation and firm–level productivity effects. The standard

[8] On the surface τ_a looks similar to the revenue distortion τ_y of Hsieh and Klenow (2009), however it plays a different role in our setting. τ_a is a wedge between potential and actual physical productivity, reflecting the effect of lower schooling investments. It is thus part of the definition of TFPQ, whereas τ_y would be part of the definition of TFPR. In fact, in our model there are no revenue distortions as defined by Hsieh and Klenow (2009).

deviation of log TFPR is three times higher in India, and average TFPQ is about 25% lower. The standard deviation of TFPQ is also higher in India, due to poorer selection and thus an extended left tail of low productivity entrepreneurs that only find it profitable to produce when financial frictions are severe (hence when input prices are lower). These are the two main salient features highlighted by Hsieh and Klenow (2009) when comparing the empirical distributions of TFPR and TFPQ in the U.S. and India (and China). Our model is consistent with this evidence.

3.4 Aggregate Productivity

Now we provide a connection between the distributions of TFPR and TFPQ, and aggregate TFP. The final good sector admits an aggregate production function (see Appendix D):

$$Y = \mathrm{TFP}\left(K^{\alpha} L^{1-\alpha}\right)^{\gamma},$$

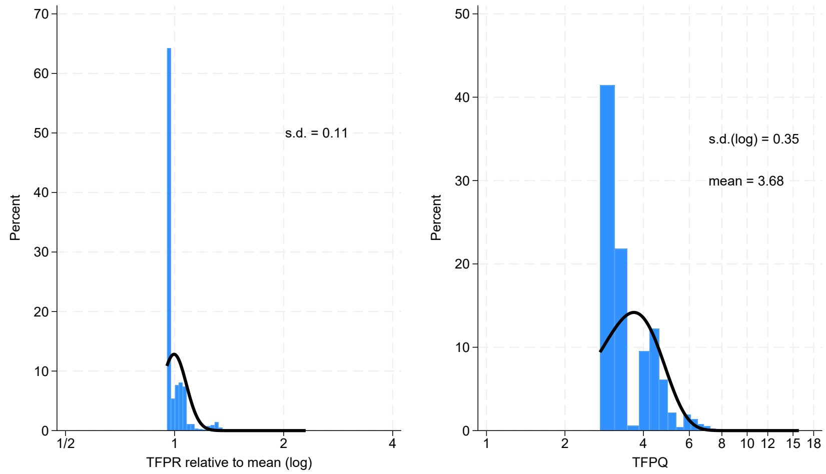
where $Y \equiv \int_M y d\Psi$, $K \equiv \int_M k d\Psi$, and $L \equiv \int_M l d\Psi$, with M the set of individual states selecting into entrepreneurship, Ψ the cross-sectional distribution over individual states, and $m \equiv \int_M d\Psi$ the entrepreneurship rate. TFP is an aggregate of individual physical productivities and distortions

$$\mathrm{TFP} \equiv \frac{\left[\int_M a^* \left(\frac{1-\tau_a}{(1+\tau_k)^{\alpha\gamma}}\right)^{\frac{1}{1-\gamma}} d\Psi\right]^{1-\gamma(1-\alpha)}}{\left[\int_M a^* \left(\frac{1-\tau_a}{(1+\tau_k)^{1-\gamma(1-\alpha)}}\right)^{\frac{1}{1-\gamma}} d\Psi\right]^{\alpha\gamma}}. \tag{20}$$

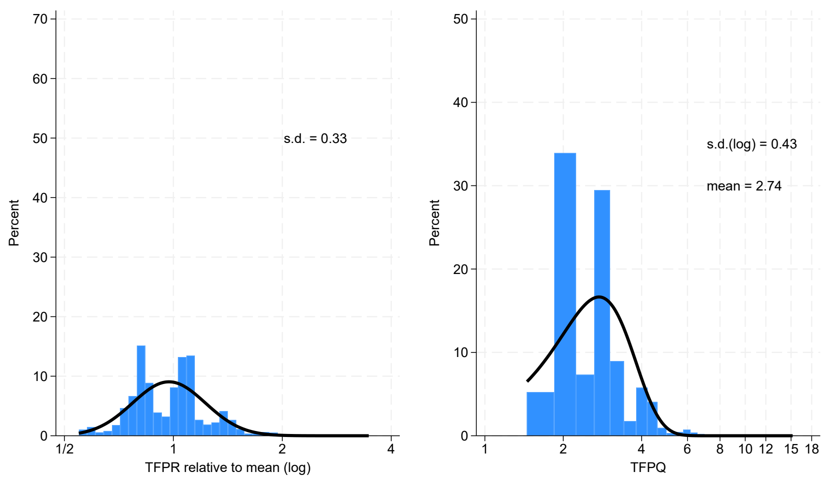
We can rewrite it as

$$\mathrm{TFP} = \int_M \left(\mathrm{TFPQ}\frac{\overline{\mathrm{TFPR}}'}{\mathrm{TFPR}'}\right)^{\frac{1}{1-\gamma}} d\Psi, \tag{21}$$

with $\mathrm{TFPR}' \equiv \mathrm{TFPR}\,(1+\tau_k)^{\alpha(\gamma-1)} \propto\ (1+\tau_k)^{\alpha\gamma}$, and where $\overline{\mathrm{TFPR}}'$ is a geometric average of marginal products of capital and labor which normalizes the weights on TFPQ. The presence of decreasing returns to scale ($\gamma < 1$) makes these weights differ very slightly from TFPR. Expression (21) is very similar to the one obtained



(a) TFPR and TFPQ in the U.S.



(b) TFPR and TFPQ in India

Figure 1: Revenue and physical productivity distributions in the model

in Hsieh and Klenow's (2009) accounting framework. However, here distortions do impact TFPQ.

To build intuition for these effects, consider the case in which $(a^*)^{1-\gamma}$, $(1-\tau_a)$, and $(1+\tau_k)$ are jointly log-normally distributed across firms. This yields:

$$\log \mathrm{TFP} = \underbrace{(1-\gamma)\log m}_{\text{Specialization}} + \underbrace{(1-\gamma)\log \mathbb{E}_M\left[\mathrm{TFPQ}^{\frac{1}{1-\gamma}}\right]}_{\text{Firm–level productivity}} - \underbrace{\frac{1}{2}\frac{\alpha\gamma\left(1-\gamma+\alpha\gamma\right)}{1-\gamma}\mathrm{var}_M\left(\log\left(1+\tau_k\right)\right)}_{\text{Capital misallocation}}, \quad (22)$$

where the moments are conditional on the states in M, and firm–level productivity is now further decomposed as

$$\log \mathbb{E}_M\left[\mathrm{TFPQ}^{\frac{1}{1-\gamma}}\right] = \underbrace{\log \mathbb{E}_M a^*}_{\text{Potential productivity}} + \underbrace{\log \mathbb{E}_M\left(1-\tau_a\right)^{\frac{1}{1-\gamma}}}_{\text{Schooling under-investment}} + \underbrace{\mathrm{cov}_M\left(\log a^*, \log\left(1-\tau_a\right)\right)}_{\text{Schooling misallocation}}. \quad (23)$$

The first term in (22) is the gain from *specialization*. Due to decreasing returns to scale, aggregate productivity rises when output is produced by a larger number of smaller firms. The remaining terms illustrate two channels through which firm-level distortions induced by credit frictions reduce the aggregate TFP. *Capital misallocation*, associated with the dispersion in τ_k, is due to the lack of equalization of marginal products of capital across firms. This is the effect normally emphasized in the literature, e.g. Hsieh and Klenow (2009).

Our main focus is on the consequences for *firm–level productivity*. Here τ_a lowers average firm-level physical productivity, by introducing a gap between actual (TFPQ) and potential $((a^*)^{1-\gamma})$ physical productivities, reflecting lower human capital investments by constrained entrepreneurs. This generates further aggregate TFP effects from financial frictions, beyond capital misallocation.

Equation (23) decomposes the firm–level productivity effect into three elements. *Potential productivity* is determined by the selection of households into entrepreneurship, and thus by the misallocation of talent.[9] In addition, it also reflects changes in entrepreneurial human capital investments due to changes in prices. *Schooling under-investment* reflects the fact that financial frictions discourage entrepreneurial investments in human capital. *Schooling misallocation* stems from the interaction between selection into entrepreneurship and human capital investments of entrepreneurs. A negative covariance between potential productivity and $1 - \tau_a$ decreases aggregate TFP, since in this case entrepreneurs with the best talent also face the largest disincentives to schooling investment, and therefore experience the largest productivity decline relative to potential.

4 Schooling and entrepreneurial productivity: some evidence

A key ingredient of our theory is that human capital accumulation by entrepreneurs increases firm-level productivity. Two related implications are that more educated entrepreneurs run larger firms, and that they enjoy higher earnings. Cross-sectional heterogeneity in schooling, our model suggests, may stem from differences in either learning ability, entrepreneurial ability, or wealth. Our main goal in this section is to present corroborating evidence showing that human capital is indeed positively associated to entrepreneurial outcomes. We abstract from the sources of variation in schooling, and an empirical assessment of causality. Our second goal is to obtain empirical targets for our calibration exercise.

Evidence is available for the U.S. from the NLSY79 (Bureau of Labor Statistics, U.S. Department of Labor, 2014). Our starting point is Levine and Rubinstein's (2017) sample of individuals aged 25 and over between 1982 and 2012, with available information on employment status. Differently from them, we restrict attention to the representative sample, and to self-employed individuals working full-time, full-year. We equate self-employment with entrepreneurship. We measure firm size with the number of workers, which includes the self-employed business owner in addition

[9]To the extent that all moments in (22) are conditional on the set of entrepreneurs M, they are all affected by misallocation of talent.

Dependent variable:	number of workers (log)		paid employees dummy	earnings (log)
	(1)	(2)	(3)	(4)
years of schooling	0.0400	0.0387	0.0327	0.109
	(0.089)	(0.112)	(0.002)	(0.000)
gender and non-white	no	yes	yes	yes
cubic experience	no	no	no	yes
industry dummies	yes	yes	yes	no
number of observations	990	990	990	3,497
R^2	0.184	0.189	0.093	0.169

Note: All regressions include year dummies. Industry dummies are for the main job, at the one-digit level (2000 Census). P–values in parenthesis based upon robust standard errors, clustered at the individual level.

Table 1: Schooling, firm size, and earnings premiums

to all paid employees, and is available every other year starting in 2002. Earnings are CPI-deflated yearly wages plus income from business. Years of schooling is the number of years corresponding to the highest grade attained.

Table 1 contains the results. We start with the effect of entrepreneurial schooling on firm size. Our baseline regression in the first column is a straightforward firm size regression, with entrepreneurial schooling as the key determinant. Schooling has a significant impact on firm size, which is expected to increase by 4% for each additional year of schooling. Adding gender and race controls makes the coefficient on schooling barely insignificant at 10%. Many self-employment firms in our sample, however, only employ the business owner. This is consistent with Levine and Rubinstein (2017): these self-employed individuals run relatively basic unincorporated businesses, and tend to be much less educated than those that incorporate. This suggests that schooling might have a more significant effect along the extensive margin of firm size. In regression (3) we therefore run a simple linear probability model along the lines of regression (2), except that the dependent variable is now a dummy for whether the business has paid employees. In this case the coefficient on schooling is indeed positive and highly significant, suggesting the extensive margin to be the most significant and robust effect of entrepreneurial human capital on business size.

Regression (4) is a basic Mincerian earnings regression among entrepreneurs.

We obtain a significant coefficient of about 11%. This magnitude is in line with what literature has found for workers and self-employed confounded (Card, 1999), although we make no attempt here to control for learning ability. Taken together, we interpret these findings as providing support for the main ingredient in our model. They suggest schooling investments do play an important role for entrepreneurial outcomes, consistent with more educated entrepreneurs being more productive.

Our findings are related to a recent literature, following Bloom and Reenen (2007), which has been documenting a strong association between cross–firm differences in management practices and firm–level productivity levels. This literature has also uncovered some of the reasons behind the heterogeneity in firm management, like differences in manager's (or employees more generally) ability, but also differences in the extent of product market competition, and the interaction between firm–level "hard" technological factors and aggregate–level factors such as contract enforcement quality, social capital, or institutions favoring dynastic management, which may limit the scope of decision-making delegation and ultimately firm growth of more productive firms (Bloom et al., 2016). Our analysis abstracts from delegation, hence from the latter set of factors. In relation to this literature, our focus is on the role of inside management's ability for firm–level productivity, namely how it depends on formal schooling and its determinants.

5 Calibration

Our baseline strategy is similar to Buera and Shin's (2013) and several others, in the sense that we first calibrate the model economy to the U.S. and then vary the financial friction λ, holding the remaining parameters constant, in order to match India's ratio of external finance to output. We call this the *benchmark* India calibration.

We consider an alternative *schooling* calibration where we also allow the productivity of the schooling sector A_h to be lower than 1 in India, in order to match India's much lower average years of schooling compared to the U.S. This exercise is relevant in light of Section 3.3.2, suggesting larger schooling disincentives with $A_h < 1$. We view $A_h < 1$ as representing India-U.S. differences in school quality not captured by our modelling of current/within-cohort expenditures. These could be due to differences in school infrastructure (like buildings), to differences in school institutions (like school accountability and autonomy, public vs private mix), or even to differences in health infrastructure. All these factors affect how productive schooling inputs are in generating human capital. See Hanushek and Woessmann (2011) and Woessmann (2016) for a discussion.

Table 2 has the baseline parameters. The first-order Markov chain governing abilities is obtained from the discretization of a VAR(1) in logs where

$$\ln\left(z_{t+1}/\bar{z}\right) = \rho_z \ln\left(z_t/\bar{z}\right) + \varepsilon^z_{t+1},$$
$$\ln\left(x_{t+1}/\bar{x}\right) = \rho_x \ln\left(x_t/\bar{x}\right) + \varepsilon^x_{t+1},$$

and the disturbances are normally distributed with variance-covariance matrix

$$\mathbf{\Sigma} = \begin{pmatrix} \sigma_z^2 & \sigma_{zx} \\ \sigma_{zx} & \sigma_x^2 \end{pmatrix}.$$

We employ the procedure described by Tauchen and Hussey (1991), with 15 states for entrepreneurial ability and 4 states for learning ability.

One model period is 30 years. Individuals start life at age 6. From age 6 until age 36 (childhood) is the period when schooling and early working in the labor market take place. From age 36 until retirement age 66 (adulthood) is the period when the main economic activity, entrepreneurship or working for a wage, takes place.

Some parameters are calibrated externally to the model. These are in the top block of Table 2. The coefficient of relative risk aversion belongs to the interval of available estimates, and is a standard value in quantitative analysis, as is the rate of physical capital depreciation. The parameters governing the income share of capital (α) and the income share of entrepreneurial income (γ) are also standard in models of entrepreneurship (see for example Atkeson and Kehoe, 2005, who base their calibration on a survey of direct estimates, as well as Restuccia and Rogerson, 2008, Buera and Shin, 2013, and Midrigan and Xu (2014)).[10] We set the autocorrelation

[10] Given decreasing returns to scale, income accrues to capital, labor, and the entrepreneurial input. We attribute the latter to capital and labor incomes, in shares α and $1-\alpha$ respectively. We therefore equate α to the aggregate capital income share value.

Table 2: Benchmark calibration

Parameter	Value	Target	Model	Data
External calibration				
σ	1.0	direct estimates		
δ	0.844	yearly depreciation rate of 6%		
α	1/3	capital income share		
γ	0.85	direct estimates		
ρ_z	0.72	intergenerational correlation of IQ scores		
$\bar{s}$	2/3	up to 20 years of formal schooling (ages 6 to 26)		
Internal calibration				
β	0.205	yearly real interest rate	0.036	0.04
$\bar{z}$	26.0	average years of schooling among entrepreneurs	14.2	13.6
ξ	0.965	average years of schooling among workers	13.7	14.1
σ_z	0.138	earnings share of top 5%	0.36	0.35
$\bar{x}$	0.837	Mincerian returns to schooling among entrepreneurs	0.117	0.109
η	0.67	output share of schooling expenditures	0.082	0.105
$\bar{l}$	2.89	output share of teacher and staff compensation	0.03	0.05
ψ	0.6	average labor earnings at age 46 over average at age 26	1.73	1.75
ρ_x	0.46	intergenerational correlation of entrepreneurship	0.27	0.32
σ_x	0.27	employment share of top 5% establishments	0.59	0.57
σ_{zx}	-0.107	ratio of median earnings (entrepreneurial over labor)	1.15	1.10
ϕ	0.0041	share of household credit in total external finance	0.18	0.19
$\lambda_{\text{U.S.}}$	35.0	ratio of external finance to output (U.S.)	2.42	2.91
λ_{INDIA}	1.345	ratio of external finance to output (India)	0.46	0.46

coefficient of learning ability to the intergenerational correlation coefficient of IQ scores reported by Bowles and Gintis (2002), between the average parental and the average offspring IQ scores. Finally, we impose an upper bound on schooling time corresponding to 20 years of formal schooling.

The remaining 14 parameters are chosen in order to minimize the sum of squared percentage deviations of 14 data moments from their model analogues. The bottom block of Table 2 shows the values for these parameters, as well as how the model's moments compare to the data. As is common in this type of analysis, we identify each parameter with a moment which we believe is particularly helpful in identifying it, although in the end all parameters are jointly determined through a fairly complex system of nonlinear equations.

When computing the model moments we assume a survey protocol that mimics the data, where individuals are interviewed every year while labor force participants. Due to schooling time, young workers are therefore interviewed less often than either adult workers or entrepreneurs. We also take one entrepreneurial firm in the model as corresponding to an establishment in the data.[11]

We comment on each of the moments we have selected. A yearly real interest rate of 4% is roughly between the real return on riskless bonds and the real return on equity over a long horizon. Based upon the NLSY79 data of Section 4, we compute three summary statistics: around 14 average years of schooling for both self-employed (our notion of entrepreneurs) and salaried workers and a ratio between the median annual earnings among self-employed to the median across salaried workers of 1.10. The latter moment is key in identifying a slightly negative covariance between innovations to learning and entrepreneurial abilities. Meaning that households with high learning ability tend to have a slight disadvantage at entrepreneurship. Otherwise entrepreneurs would have much higher earnings relative to workers, compared to the data. We also use our estimate of the Mincerian returns to schooling among entrepreneurs from Table 1.

For the output shares of (public and private) schooling expenditures and teacher and staff compensation we use the same numbers as Erosa et al. (2010). These are based upon total expenditure data for 1990 to 1995 from the U.S. Department of Education, together with an estimate of the share of teacher and staff compensation from the OECD. The ratio of average labor earnings at age 46 over age 26 comes from Figure 1 of Kambourov and Manovskii (2009). It is based upon the PSID (Panel Study of Income Dynamics) and refers to the cohort entering the labor market in 1968. The intergenerational correlation of entrepreneurial occupation is reported by Dunn and Holtz-Eakin (2000), and corresponds to the fraction of sons of self-employed fathers in the NLS (National Longitudinal Surveys) who were themselves

[11] This is in line with the related literature, namely Buera et al. (2011), Buera and Shin (2013), and Midrigan and Xu (2014). Our choice of calibration targets reflects this view. We acknowledge the caveat that, in the data, there exist multi-establishment firms, and more importantly firms whose ownership does not coincide with management, or more generally self-employment.

self-employed at some point in the sample.

The employment share of the top 5% establishments is reported by Henly and Sanchez (2009), based upon the U.S. Census County Business Pattern series. This figure is across establishments in all sectors of activity in the year 2006. The earnings share of the top 5% comes from Díaz-Giménez et al. (2011) and is based on the Survey of Consumer Finances.

The ratio of total external finance (including private credit) to output in the U.S. was obtained from the 2013 update of the Beck et al. (2000) financial indicators database. We adjusted the reported stock market capitalization by the average book-to-market ratio, following Buera et al. (2011). Our number is the average over the years 1990 to 2011. Our other financial market indicator is the share of household credit in total external financing. We obtained it as the product between the share of household credit in total credit in 2005 from the International Monetary Fund (2006), and the share of total credit in total external financing from the 2013 update of the Beck et al. (2000) data set, again averaged over the years 1990-2011.

For India's benchmark calibration, a value of $\lambda_{\text{INDIA}} = 1.345$ allows us to match exactly India's ratio of external finance to output, obtained as described previously for the U.S. For India's schooling calibration we obtain $\lambda_{\text{INDIA}} = 1.291$, and $A_{h,\text{INDIA}} = 0.334$. With these parameters we can match exactly India's ratio of external finance to output and average of 5.95 years of schooling.

We now comment on our reliance on a two-period overlapping-generations model in order to quantify the effects of financial frictions. Buera et al. (2011) discuss the potential pitfalls of such approach. They find that a two-period version of their multi-period model understates the role of self-financing, and thus overstates the role of credit frictions. Their argument is that a multi-period environment is therefore needed in order to allow firms to grow out of the financial constraints, given persistent productivity levels.[12] Although we acknowledge this would be ideal, such approach is unfortunately very costly here: the multi-period nature of both human capital accumulation (by the child) and entrepreneurship (by the parent), together with the

[12]Implicit in this reasoning is a problem of lack of time-aggregation, i.e. a properly calibrated 30 year model like we have here might not necessarily correspond to the time aggregation of a higher frequency model.

		Y		K/Y		TFP	
		Model	Data	Model	Data	Model	Data
U.S.		1.00	1.00	2.16	2.99	1.00	1.00
India	bench	0.46	0.08	1.57	1.93	0.73	0.26
	school	0.05		1.74		0.51	

Table 3: Macroeconomic aggregates

fact that our model features two-dimensional abilities, would imply a large increase in the state-space.[13] At the same time we believe our dynastic environment offers a reasonable compromise, and it's not clear to us the usual concerns associated with two-period models apply. We note that the two-period model Buera et al. (2011) rely upon features non-altruistic agents, born with no wealth, and leaving no bequests. In contrast, we work with a dynastic environment with full altruism and unrestricted bequests. These inter-generational links, together with the fact that ability is persistent over generations, give opportunity for high-ability dynasties to grow out of the financial constraints in our two-period model. Further, in our model agents may also self-finance within a generation, by cutting back on entrepreneurial schooling investments - the central mechanism we emphasize.

6 Quantitative Assessment

Consider the quantitative implications of frictions in the model, for the U.S. and the two India calibrations, benchmark (only λ differs from the U.S.) and schooling (both λ and A_h differ from the U.S.). Table 3 looks at the implications for aggregate output, capital-output ratios, and aggregate TFP, both in the model and in the data. Our data source is version 8.1 of the Penn World Tables (Feenstra et al., 2015). Appendix E describes in detail the mapping between model and data.

The model produces significant differences in macro aggregates. The magni-

[13] Mestieri et al. (2017) do consider multi-period decisions in a dynastic framework similar to ours in spirit, however ability differences in their case are only along the learning ability dimension. They focus on several interesting implications of financial frictions over the life cycle, namely the timing of entry into entrepreneurship and firm size dynamics, for which a multi-period model is necessary.

tudes are smaller than in the data for the benchmark (*bench*) calibration, but much closer for the schooling (*school*) calibration. The measured India-U.S. TFP difference in the data is $100 - 26 = 74\%$. This difference is $100 - 73 = 27\%$ under the benchmark calibration, i.e. $27/74 = 36\%$ of the actual difference. Regarding aggregate output, the model accounts for 59% of the actual 92% difference. Under the schooling calibration, it accounts for 68% of the actual TFP difference, and generates an output difference about as large as in the data.[14]

Table 4 provides a decomposition of the model-implied TFP loss in India relative to the U.S. Based upon the intuition provided by the lognormal case in (23), we perform a counterfactual decomposition of model TFP from (20). We now treat proxy wedges and potential productivity as structural objects, and compute the following sequence of counterfactual experiments for each calibration: (i) replace m with $m_{\text{U.S.}}$ (specialization), (ii) replace τ_a with its mean (schooling misallocation), (iii) set $\tau_a = 0$ (schooling under-investment), (iv) set $\tau_k = 0$ (capital misallocation). Potential productivity is the residual effect. This exercise is very much along the lines of the lognormal one, with the main difference that we use the model-implied distribution of proxy wedges and potential productivities, rather than relying on the potentially restrictive joint log-normality assumption (e.g. capital wedges only matter through the variance in (22)).

According to this decomposition, *specialization* contributes negatively to the U.S.-India TFP difference, since the entrepreneurship rate is higher in India. The other two terms, *firm-level productivity* and *capital misallocation*, respectively contribute to a 23% and a 17% TFP difference under the benchmark calibration. Our model produces TFP losses from capital misallocation which are a bit higher than those in Midrigan and Xu's (2014) model calibrated to Korea, which tend to stay below 10%, but significantly lower than those reported by Hsieh and Klenow (2009), which can be as high as 60%. Like in Midrigan and Xu (2014), entrepreneurs adjust to the presence of financial frictions by relying more on self-financing. In our model, this happens while entrepreneurs cut back on schooling investments

[14] The reason the latter calibration delivers output differences in line with the data in spite of lower TFP differences (and similar capital-output ratios) is that human capital stock differences turn out to be larger in the model than the PWT8.1 estimates we rely upon to compute TFP - see Appendix E - even if we do match average years of schooling differences.

and spend a larger fraction of their early lives working for a wage, mitigating the quantitative role for capital misallocation.

The contribution of *firm-level productivity* is further decomposed into three terms. *Potential productivity* is on average higher in India, since input prices are lower, in spite of a worse ability selection into entrepreneurship. Lower input prices give incentives for unconstrained entrepreneurs to expand their production scale, and hence invest more in education. A lower interest rate also encourages entrepreneurs to invest more in education, for given production scale. This term therefore contributes negatively to the model-implied U.S.-India TFP difference. However, *schooling under-investment* is more important in India, leading to a 22.7% TFP loss. Finally, there is also a significantly higher degree of *schooling misallocation* in India: the most talented entrepreneurs are the ones cutting back the most in terms of education, and this effect entails a 17.6% TFP loss. Taken together, schooling under-investment and schooling misallocation are the most important drivers of India's model-implied TFP loss.

The firm-level productivity effect is significantly larger under the schooling calibration, accounting for a 49.3% TFP loss in India. One of the main effects comes from what is now a 6.8% loss in potential productivity. This is due in part due to a purely mechanical effect, but also to an amplification of lower schooling investments in India. Our emphasis is on the latter effect. Higher financial frictions in India play a more significant role when A_h is low, due to lower human capital accumulation by entrepreneurs: schooling under-investment contributes to a TFP loss which is now 13 percentage points larger, whereas schooling misallocation is actually slightly lower.

It is possible to obtain a back-of-the-envelope figure for India's TFP loss under the schooling calibration which nets out the mechanical effect of a lower A_h. This effect amounts to a $1 - A_{h,\text{INDIA}}^{1-\gamma} = 15.2\%$ TFP loss. Without it, the potential productivity loss term can be approximated to $6.8 - 15.2 = -8.4\%$, actually a gain.[15] The total TFP loss in India would then still be 35.3%, still significantly

[15]Potential productivity captures direct, as well as some indirect effects of $A_{h,\text{INDIA}} < 1$: h^* decreases induced by lower schooling inputs, h^* increases induced by lower input prices, and a^* decreases due to a more adverse talent misallocation. The fact that we obtain a gain means the effect of lower input prices is dominant, similarly to the benchmark calibration.

TFP term	% Loss India relative to U.S.			
	bench		school	
Specialization	–14.8		–21.6	
Firm-level productivity	+22.9		+49.3	
Potential productivity		–21.0		+6.8
Schooling under-investment		+22.7		+35.7
Schooling misallocation		+17.6		+15.4
Capital misallocation	+17.1		+18.1	
TFP total	26.6		49.5	

Note: Total TFP is recovered from the product of the partial effects.

Table 4: Aggregate TFP loss decomposition

higher than the 26.6% we obtain under the benchmark calibration. The intuition behind these larger effects follows from our discussion in Section 3.3.2. A lower A_h effectively weakens the self-financing channel for future entrepreneurs, by lowering first-period labor earnings. This makes schooling investments even more costly, thus amplifying schooling under-investments.

It is worth to very briefly compare our TFP results with the related quantitative literature. Buera and Shin (2013) and Buera et al. (2011) are among the closest papers. Going from the perfect credit markets to financial autarky, the TFP losses are estimated to be 24% in the former (which features exogenously-given firm-level taxes/subsidies) and 36% in the latter (which features cross-sectoral variation in fixed costs). Midrigan and Xu (2014) also estimate TFP losses up to 26% from shutting down external finance relative to a Korean calibration, in a framework with entry into entrepreneurship and the possibility of technology adoption. Our estimates are therefore on the higher end of what the literature has found, especially considering the role of schooling productivity. The comparison, however, is not straightforward, given different model features, calibrations, and range of variation of the severity of financial frictions. Financial frictions may quite plausibly generate larger TFP effects if schooling investments were to be combined with the ingredients emphasized by these different papers.

The next two sets of results provide a simple illustration of how well our model does in matching certain micro–level production and schooling outcomes. Table 5

		ent.rate		avg. firm size	
		Model	Data	Model	Data
U.S.		4.8	8.7	1.00	1.00
India	bench	12.0	48.6	0.38	0.29
	school	17.7		0.26	

Table 5: Entrepreneurship rate and average firm size

focuses on production. It displays the rate of entrepreneurship and the average firm size (relative to the U.S).

The rate of entrepreneurship in the U.S. is based upon the data from Section 4. For India, we rely on information from Ministry of Statistics and Programme Implementation (2014).[16] Entrepreneurs are household heads reporting to be self-employed, and workers includes both salaried and casual labor. We report numbers for ages 15 to 59, and across all genders, sectors, and regions.

In the data, our measure of size is the number of paid employees.[17] Once again, we take the data counterpart of an entrepreneurial firm in the model to be an establishment. For the U.S., the evidence comes from Henly and Sanchez (2009), based on the Census Bureau's 2006 County Business Pattern Series. They report an average of 15 employees per establishment across all sectors of activity (their Figure 1). For India, we rely on the Fifth Economic Census data from Ministry of Statistics and Programme Implementation (2008), which is for 2005. The data is available for all sectors of activity across all Indian states, in both urban and rural settings. It provides the same type of information (i.e. establishment and worker counts by establishment size groups, for establishments with hired workers) as the County Business Pattern Series in the U.S. This allows us to apply the same method as Henly and Sanchez (2009) to obtain approximations to the relevant moments of the size distribution in India from the establishment and worker counts, and ensures comparability across the two countries. We obtain an average of 4.38 employees

[16] All Indian data used throughout the paper is available at https://www.mospi.gov.in/.

[17] We use the total firm-level labor input as the model counterpart. Unfortunately our model does not distinguish between the number of workers and the quantity of human capital employed. To partially address this issue, we equate the number of workers employed by a firm to $\max\{l/\bar{h}^w, 1\}$, where $\bar{h}^w$ is the average level of human capital per worker in the whole economy.

		aggregate		workers		entrepreneurs	
		Model	Data	Model	Data	Model	Data
U.S.		13.72	14.03	13.70	14.07	14.17	13.63
India	bench	13.33	5.95	13.50	6.11	12.13	5.79
	school	5.95		6.53		3.25	

Table 6: Years of schooling

per establishment in India, implying a India-U.S. ratio of 0.29 in the data.

Consistently with the data, our model generates more entrepreneurs in India, operating on average at a smaller scale. The main mechanism driving the higher rate of entrepreneurship is the drop in input prices, which encourages lower ability individuals to engage in production. The magnitude, however, is much lower than in the data.[18]

Overall, we can say that the model delivers firm-size distribution differences which are consistent with the data. The model accounts for 76% of the 2/3 India-U.S. difference in average firm size under the benchmark calibration, and nearly matches that difference under the schooling calibration.

Table 6 looks at average years of schooling. The U.S. data, aggregate and by occupation, are the NLSY79 used in our calibration. For India, we again rely on Ministry of Statistics and Programme Implementation (2014), and focus on population aged 15 to 59 between 2011 and 2012, across all sectors and regions, including regular and casual workers.

The model produces lower schooling levels in India independent of occupation, but by much less than in the data under the benchmark calibration. The main reason the model is unable to deliver a larger effect is that the interest rate is lower in India, which incentivizes larger schooling investments.[19]

[18] If we exclude agriculture from the data, in an extreme attempt to deal with the large importance of subsistence farming in India, we still obtain an entrepreneurship rate of 37.8%. Such high self-employment rates are most likely an artifact: the data counts helpers in family business as self-employed, in addition to own-account workers and employers.

[19] The cross-country variation in schooling time is also lower than in the data in Erosa et al. (2010), see their Figure 3. They argue that their human capital accumulation environment, same as ours, tends to understate cross-country schooling time differences, while at the same time generating large quality differences. Our model produces similar implications under the benchmark calibration.

The schooling calibration matches the average years of schooling in India by design. Schooling is now much lower for both workers and entrepreneurs. The effect is significantly more pronounced for entrepreneurs in the model, more so than in the data. A larger entrepreneurial response does stem from our central mechanism, with entrepreneurial under-investment amplified by lower schooling sector productivity. Such a counterfactually large response of entrepreneurial schooling in the model compared to the data suggests that the distortions and the induced TFP effects associated with this calibration are possibly too extreme. We therefore view the plausible TFP losses in India to be somewhere in the 36% to 68% range.

7 Concluding Remarks

We investigate the aggregate productivity effects of financial frictions, in an environment where frictions impact both firm-level investment decisions, and household-level schooling decisions. We show that, in anticipation of the effect credit constraints have on their future business activity, entrepreneurs under-invest in schooling. Further, this behavior is more pronounced among the most able entrepreneurs, generating a misallocation of schooling investments. Both effects are shown to produce important aggregate productivity losses, ranging from 36% to 68% of the U.S.-India aggregate productivity difference. These findings imply that schooling distortions are a major source of productivity differences. Our research suggests educational policies, such as tuition subsidies or public provision of schooling, may have significant productivity effects. Analyzing the role of such policies in reducing misallocation and improving production outcomes is a natural and interesting direction for further work.

References

Amaral, P. S., and E. Quintin (2010) "Limited enforcement, financial intermediation, and economic development: A quantitative assessment," *International Economic Review* 51(3), 785–811

Atkeson, A., and P. Kehoe (2005) “Modeling and measuring organizational capital,” *Journal of Political Economy* 113, 1026–1053

Barro, R., and J.-W. Lee (1993) “International comparisons of educational attainments,” *Journal of Monetary Economics* 32, 363–94

Bartelsman, E., J. Haltiwanger, and S. Scarpetta (2013) “Cross-Country Differences in Productivity: The Role of Allocation and Selection,” *American Economic Review* 103(1), 305–34

Beck, T., A. Demirgüç-Kunt, and R. Levine (2000) “A new database on financial development and structure,” *World Bank Economic Review* 14, 597–605

Bhattacharya, D., N. Guner, and G. Ventura (2013) “Distortions, endogenous managerial skills and productivity differences,” *Review of Economic Dynamics* 16(1), 11–25

Bloom, N., and J. V. Reenen (2007) “Measuring and Explaining Management Practices Across Firms and Countries,” *The Quarterly Journal of Economics* 122(4), 1351–1408

Bloom, N., R. Sadun, and J. V. Reenen (2016) “Management as a Technology?,” NBER Working Papers 22327, National Bureau of Economic Research, Inc

Bowles, S., and H. Gintis (2002) “The Inheritance of Inequality,” *Journal of Economic Perspectives* 16(3), 3–30

Buera, F. J., J. P. Kaboski, and Y. Shin (2011) “Finance and development: A tale of two sectors,” *American Economic Review* 101(5), 1964–2002

Buera, F. J., and Y. Shin (2013) “Financial frictions and the persistence of history: A quantitative exploration,” *Journal of Political Economy* 121(2), 221 – 272

Bureau of Labor Statistics, U.S. Department of Labor (2014) “National Longitudinal Survey of Youth 1979 cohort,” Produced and distributed by the Center for Human Resource Research (CHRR), The Ohio State University. Columbus, OH, 1979-2012 (rounds 1-25)

Card, D. (1999) “The causal effect of education on earnings,” in O. Ashenfelter and D. Card, eds., *Handbook of Labor Economics*, volume 3 of *Handbook of Labor Economics*, chapter 30, pp. 1801–1863, Elsevier

Caselli, F. (2005) “Accounting for cross-country income differences,” in P. Aghion and S. Durlauf, eds., *Handbook of Economic Growth*, volume 1 of *Handbook of Economic Growth*, chapter 9, pp. 679–741, Amsterdam, New York and Oxford: Elsevier Science, North Holland

Castro, R., G. L. Clementi, and G. MacDonald (2004) “Investor protection, optimal incentives, and economic growth,” *Quarterly Journal of Economics* 119(3), 1131–1175

——— (2009) “Legal institutions, sectoral heterogeneity, and economic development,” *Review of Economic Studies* 76(2), 529–561

Clausen, A., and C. Strub (2020) “Reverse Calculus and nested optimization,” *Journal of Economic Theory* 187(C)

Cole, H. L., J. Greenwood, and J. M. Sanchez (2016) “Why Doesn’t Technology Flow From Rich to Poor Countries?,” *Econometrica* 84, 1477–1521

Córdoba, J. C., and M. Ripoll (2013) “What explains schooling differences across countries?,” *Journal of Monetary Economics* 60(2), 184–202

Dunn, T., and D. Holtz-Eakin (2000) “Financial Capital, Human Capital, and the Transition to Self-Employment: Evidence from Intergenerational Links,” *Journal of Labor Economics* 18(2), 282–305

Díaz-Giménez, J., A. Glover, and J.-V. Ríos-Rull (2011) “Facts on the distributions of earnings, income, and wealth in the United States: 2007 update,” *Quarterly Review* 34(1), 2–31

Erosa, A., and A. Hidalgo-Cabrillana (2008) “On finance as a theory of TFP, cross-industry productivity differences, and economic rents,” *International Economic Review* 49(2), 437–473

Erosa, A., T. Koreshkova, and D. Restuccia (2010) "How important is human capital? a quantitative theory assessment of world income inequality," *Review of Economic Studies* 77(4), 1421–1449

Evans, D., and B. Jovanovic (1989) "An estimated model of entrepreneurial choice under liquidity constraints," *Journal of Political Economy* 97, 808–27

Feenstra, R. C., R. Inklaar, and M. P. Timmer (2015) "The next generation of the Penn World Table," *American Economic Review* 105(10), 3150–3182, available for download at www.ggdc.net/pwt

Foster, L., J. Haltiwanger, and C. Syverson (2008) "Reallocation, firm turnover, and efficiency: Selection on productivity or profitability?," *American Economic Review* 98(1), 394–425

Galor, O., and J. Zeira (1993) "Income distribution and macroeconomics," *Review of Economic Studies* 60(1), 35–52

Greenwood, J., J. Sanchez, and C. Wang (2013) "Quantifying the impact of financial development on economic development," *Review of Economic Dynamics* 16(1), 194–215

Hall, R. E., and C. I. Jones (1999) "Why do some countries produce so much more output per worker than others?," *The Quarterly Journal of Economics* 114(1), 83–116

Hanushek, E. A., and L. Woessmann (2011) *The Economics of International Differences in Educational Achievement*, volume 3 of *Handbook of the Economics of Education*, chapter 2, pp. 89–200, Elsevier

Henly, S. E., and J. M. Sanchez (2009) "The U.S. establishment-size distribution: secular changes and sectoral decomposition," *Economic Quarterly* 95(4), 419–454

Hsieh, C.-T., and P. J. Klenow (2009) "Misallocation and manufacturing TFP in China and India," *The Quarterly Journal of Economics* 124(4), 1403–1448

——— (2010) "Development accounting," *American Economic Journal: Macroeconomics* 2(1), 207–23

International Monetary Fund (2006) *Global Financial Stability Report: Market Developments and Issues*, Washington, DC: International Monetary Fund

Jeong, H., and R. Townsend (2007) "Sources of TFP growth: occupational choice and financial deepening," *Economic Theory* 32(1), 179–221

Kambourov, G., and I. Manovskii (2009) "Accounting for the changing life-cycle profile of earnings," University of Toronto

Kehoe, T. J., and D. K. Levine (1993) "Debt-constrained asset markets," *Review of Economic Studies* 60(4), 865–888

Klenow, P. J., and A. Rodríguez-Clare (1997) "The neoclassical revival in growth economics: Has it gone too far?," in *NBER Macroeconomics Annual 1997*, volume 12, pp. 73–103, Cambridge, MA: MIT Press

Levine, R., and Y. Rubinstein (2017) "Smart and Illicit: Who Becomes an Entrepreneur and Do They Earn More?," *The Quarterly Journal of Economics* 132(2), 963–1018

Lochner, L. J., and A. Monge-Naranjo (2011) "The Nature of Credit Constraints and Human Capital," *American Economic Review* 101(6), 2487–2529

Manuelli, R. E., and A. Seshadri (2014) "Human Capital and the Wealth of Nations," *American Economic Review* 104(9), 2736–62

Mestieri, M., J. Schauer, and R. Townsend (2017) "Human Capital Acquisition and Occupational Choice: Implications for Economic Development," *Review of Economic Dynamics* 25, 151–186

Midrigan, V., and D. Y. Xu (2014) "Finance and Misallocation: Evidence from Plant-Level Data," *American Economic Review* 104(2), 422–58

Ministry of Statistics and Programme Implementation (2008) "Fifth Economic Census 2005," Government of India, Central Statistical Office, Chapter 2, All India Report

——— (2014) "Employment and Unemployment Situation in India, July 2011-June 2012," Government of India, National Sample Survey Office, 68th round, NSS Report No. 554 (68/10/1)

Moll, B. (2014) "Productivity Losses from Financial Frictions: Can Self-Financing Undo Capital Misallocation?," *American Economic Review* 104(10), 3186–3221

Moll, B., R. M. Townsend, and V. Zhorin (2017) "Economic development, flow of funds, and the equilibrium interaction of financial frictions," *Proceedings of the National Academy of Sciences* 114(24), 6176–6184, ISSN 0027-8424

Restuccia, D., and R. Rogerson (2008) "Policy distortions and aggregate productivity with heterogeneous plants," *Review of Economic Dynamics* 11(4), 707–720

Tauchen, G., and R. Hussey (1991) "Quadrature-Based Methods for Obtaining Approximate Solutions to Nonlinear Asset Pricing Models," *Econometrica* 59(2), 371–96

Woessmann, L. (2016) "The Importance of School Systems: Evidence from International Differences in Student Achievement," *Journal of Economic Perspectives* 30(3), 3–32

Appendix

A Competitive Equilibrium

Definition. *A stationary recursive competitive equilibrium is a set of value functions $v^w(\omega, z, x)$, $v^e(\omega, z, x)$, and $v(\omega, z, x)$, together with the associated decision rules, a set of entrepreneurial households M, prices w and r, and an invariant distribution over household states Ψ such that:*

- *given prices, $v^w(\omega, z, x)$ and $v^e(\omega, z, x)$ solve problems (Pw) and (Pe), respectively, and $v(\omega, z, x)$ solves (10),*
- *the set of entrepreneur-households is defined by:*

$$M = \{(\omega, z, x) \in S \mid v^e(\omega, z, x) > v^w(\omega, z, x)\},$$

  where $S \subseteq \mathbb{R} \times \mathbb{R}_+^2$ is the individual household's state space,

- *market for labor clears:*

$$\int_M l d\Psi + \int_S s\bar{l} d\Psi = \int_{S \setminus M} h d\Psi + \int_S (1 - s) \psi h d\Psi,$$

- *market for capital clears:*

$$\int_M k d\Psi = \int_S \frac{q}{1 + r} d\Psi,$$

- *market for goods clears:*

$$\int_S c d\Psi + \int_S e d\Psi + \delta \int_M k d\Psi = \int_M a^{1-\gamma} \left(k^\alpha l^{1-\alpha}\right)^\gamma d\Psi,$$

- *distribution Ψ is invariant and defined by:*

$$\Psi\left(\hat{S}\right) = \int_S P\left(X, \hat{S}\right) d\Psi(X) \text{ for all } \hat{S} \in \mathcal{B}_S,$$

where $P : S \times \mathcal{B}_S \to [0,1]$ is a transition function generated by the decision rules and the stochastic processes for z and x, and $\mathcal{B}_S$ is the Borel σ-algebra of subsets of S.

B Profit Functions

The solution to the profit maximization problem is, for unconstrained entrepreneurs ($q \geq q^*(h,x)$, where $q^*(h,x) \equiv (1+r)k^*/\lambda$):

$$\begin{aligned}
k^* &= \left[\frac{(1-\alpha)(r+\delta)^{1-\frac{1}{(1-\alpha)\gamma}}}{\alpha w}\right]^{(1-\alpha)\frac{\gamma}{1-\gamma}} (\alpha\gamma)^{\frac{1}{1-\gamma}} a \\
l^* &= \frac{(1-\alpha)(r+\delta)}{\alpha w} k^* \\
y^* &= a^{1-\gamma}\left((k^*)^{\alpha}(l^*)^{1-\alpha}\right)^{\gamma} \\
\Pi^*(h,x) &= y^* - wl^* - (r+\delta)k^* \equiv Aa,
\end{aligned}$$

and for constrained entrepreneurs ($q < q^*(h,x)$):

$$\begin{aligned}
k^c &= \max\left\{\lambda\frac{q}{1+r}, 0\right\} \\
l^c &= \left[\frac{\gamma(1-\alpha)(k^c)^{\alpha\gamma}}{w} a^{1-\gamma}\right]^{\frac{1}{1-(1-\alpha)\gamma}} \\
y^c &= a^{1-\gamma}\left((k^c)^{\alpha}(l^c)^{1-\alpha}\right)^{\gamma} \\
\Pi^c(q,h,x) &= y^c - wl^c - (r+\delta)k^c \\
&\equiv B(q)\, a^{\frac{1-\gamma}{1-(1-\alpha)\gamma}} - (r+\delta)\frac{\lambda}{1+r}q,
\end{aligned}$$

where

$$A=\left[A_0\left(\frac{(1-\alpha)(r+\delta)}{\alpha w}\right)^{1-\alpha}\right]^{\gamma}(1-\gamma)$$

$$A_0=\left[\frac{(1-\alpha)(r+\delta)^{1-\frac{1}{(1-\alpha)\gamma}}}{\alpha w}\right]^{(1-\alpha)\frac{\gamma}{1-\gamma}}(\alpha\gamma)^{\frac{1}{1-\gamma}}$$

$$B(q)=B_0\left(q^{\alpha\gamma}\right)^{\frac{1}{1-(1-\alpha)\gamma}}$$

$$B_0=\frac{1-(1-\alpha)\gamma}{(1-\alpha)\gamma}w\left[\frac{(1-\alpha)\gamma\left(\frac{\lambda}{1+r}\right)^{\alpha\gamma}}{w}\right]^{\frac{1}{1-(1-\alpha)\gamma}}.$$

C Optimality Conditions

The first-order conditions for an interior solution to the household's problem are:[20]

$$w\left(\bar{l}+\psi h-\psi(1-s)\eta\xi\frac{h}{s}\right)u'(c)=$$
$$\beta\sum_{z',x'}\pi(z',x'|z,x)\,v_1(\omega',z',x')\,\omega_2'(q,h,x)\,\eta\xi\frac{h}{s}$$
$$\left(1-w\psi(1-s)(1-\eta)\xi\frac{h}{e}\right)u'(c)=$$
$$\beta\sum_{z',x'}\pi(z',x'|z,x)\,v_1(\omega',z',x')\,\omega_2'(q,h,x)(1-\eta)\xi\frac{h}{e}$$
$$\frac{1}{1+r}u'(c)=\beta\sum_{z',x'}\pi(z',x'|z,x)\,v_1(\omega',z',x')\,\omega_1'(q,h,x).$$

[20]Notice that v_1 is always defined at the optimum. Even though v has a kink in the wealth dimension induced by the occupational choice, the optimum will never occur at this kink. It follows that, at the optimum, v_1 is either equal to v_1^w or to v_1^e. Notice also that, with sufficient smoothness introduced by the ability shocks, which we assume, the first-order conditions are not only necessary but also sufficient for an optimum. See Clausen and Strub (2020) for a formal discussion.

D Aggregation

The individual input demands from problem (Pf′) can be written as

$$l = \frac{a^{*}\left[\frac{1-\tau_a}{(1+\tau_k)^{\alpha\gamma}}\right]^{\frac{1}{1-\gamma}}}{\int_M a^{*}\left[\frac{1-\tau_a}{(1+\tau_k)^{\alpha\gamma}}\right]^{\frac{1}{1-\gamma}} d\Psi} L \equiv \varpi_l L$$

$$k = \frac{a^{*}\left[\frac{1-\tau_a}{(1+\tau_k)^{1-\gamma(1-\alpha)}}\right]^{\frac{1}{1-\gamma}}}{\int_M a^{*}\left[\frac{1-\tau_a}{(1+\tau_k)^{1-\gamma(1-\alpha)}}\right]^{\frac{1}{1-\gamma}} d\Psi} K \equiv \varpi_k K.$$

Aggregate production is then

$$Y = \int_M y d\Psi = \text{TFP}\left(K^{\alpha} L^{1-\alpha}\right)^{\gamma}$$

where

$$\text{TFP} \equiv \int_M (1-\tau_a)\,(a^{*})^{1-\gamma}\,\varpi_k^{\alpha\gamma}\varpi_l^{(1-\alpha)\gamma} d\Psi.$$

E Mapping Between Model and Data

The aggregate production function in the data is

$$Y = \text{TFP}\left(K^{\alpha} L^{1-\alpha}\right)^{\gamma},$$

where $L \equiv h\ell N$ is the total labor input, with h being human capital per worker, ℓ the total number of workers per engaged person, and N the number of engaged persons (which includes workers and the self-employed).

We proceed in a way analogous to the related literature employing decreasing returns to scale technology (e.g. Buera and Shin, 2013) and abstract from scale effects. That is, we treat the data as if $N = 1$ for both the U.S. and India, and rewrite the aggregate production function in terms of (lowercase) variables per engaged person as

$$y = \text{TFP}\left(k^{\alpha}\,(h\ell)^{1-\alpha}\right)^{\gamma}.$$

We rely on PWT8.1 data in order to back out measured TFP for the U.S. and India. We use data for the year 2005 on current-year PPP-adjusted GDP per engaged person (variable CGDP^o divided by EMP), capital stock per engaged person (CK/EMP), and human capital stock per engaged person (variable HC), together with our parameter values for α and γ. The PWT8.1 provide human capital stock estimates by mapping average years of schooling from Barro and Lee (1993) through an exponential human capital technology specification as in Caselli (2005), using returns to schooling specific to each schooling level.

We assume that human capital per worker h, which we do not observe in PWT8.1, equals human capital per engaged person. The total labor input $h\ell$ is then computed by equating ℓ to one minus the rate of entrepreneurship from Table 5.

F Numerical Algorithm

We solve the model using value function iteration.

1. **Discretization:** Discretize ω into $\{\omega_0, \ldots, \omega_{N_\omega}\}$. We choose the upper bound and lower bounds such that increasing them further apart has a negligible effect on the solution.

   The VAR(1) process for abilities is discretized into a Markov chain using the procedure described in Tauchen and Hussey (1991).

2. **Occupational choice and production:** Solve for $\omega'(q, h, x)$ given the current guess for prices w and r.

   (i) Compute the threshold level of saving $q^*(h, x)$.

   (ii) Compute profits $\Pi(q, h, x)$.

   (iii) Compute next generation's wealth $\omega'(q, h, x)$.

3. **Saving and education:** Solve for the decision rules $e(\omega, z, x)$, $s(\omega, z, x)$, and $q(\omega, z, x)$, given $\omega'(q, h, x)$ from step 2, and given the current guess for prices.

(i) Guess value function $V^j(\omega, z, x)$ at gridpoints.

(ii) Solve for the right-hand-side of the Bellman equation:

$$V^{j+1}(\omega, z, x) = \max_{c,e,s,q} \left\{ u(c) + \beta \sum_{z',x'} \pi(z', x'|z, x) V^j(\omega'(q, h, x), z', x') \right\}$$

subject to (4)-(6).

First try an interior solution for q. If $q \geq -\lambda\phi \max\{\omega, 0\}$ then the solution has been found. Otherwise set $q = -\lambda\phi \max\{\omega, 0\}$ and find s and e subject to this constraint. V^j is approximated by a piecewise linear function for future wealth levels outside of the grid.

(iii) Iterate until $V^j(\omega, z, x) \approx V^{j+1}(\omega, z, x)$.

4. **Invariant distribution:** Approximate by simulating a large cross-section of N agents over a sufficiently large number of T periods. Decision rules are linearly interpolated over a very fine grid. The invariant distribution of individual states $\{\omega_n, z_n, x_n\}_{n=1}^{N}$ is the period T outcome.

5. **Market clearing:** Check whether the labor and capital markets clear. Compute excess demand for labor and capital from the invariant distribution as:

$$\begin{aligned} \text{EDL}(w, r) &= \frac{1}{N} \sum_{n=1}^{N} \left[\mathbb{1}_n l_n + s_n \bar{l} - (1 - \mathbb{1}_n) h_n - (1 - s_n) \psi h_n \right] \\ \text{EDK}(w, r) &= \frac{1}{N} \sum_{n=1}^{N} \left(\mathbb{1}_n k_n - \frac{q_n}{1 + r} \right), \end{aligned}$$

where $\mathbb{1}_n$ is an indicator which takes the value of 1 if household n chooses entrepreneurship and 0 otherwise, and the remaining variables indexed by n are the optimal decision rules as a function of the individual state n. Iterate on market prices until $\text{EDL}(w, r) \approx 0$ and $\text{EDK}(w, r) \approx 0$.