Economic Development and Growth in the World Economy

Rui Castro

Department of Economics and CIREQ, Université de Montréal,

Québec, Canada H3C 3J7

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Complete Mailing Address: Université de Montréal, Département de Sciences Économiques, C.P. 6128, succ. Centre-Ville, Montréal (Québec), Canada H3C 3J7.

Phone Number: (514)343-6760.

Fax Number: (514)343-7221.

Email address: rui.castro@umontreal.ca.

Abstract

This paper investigates whether technological shocks, constructed to be consistent with the observed cross-country income dispersion, are also capable of accounting for development regularities related to capital accumulation. This question is approached via a quantitative theoretical analysis of an integrated world economy model. An open economy framework constrains country heterogeneity to be consistent with international capital flows. Moreover, it enables the study of distinctively open economy development facts. The model produces time-invariant cross-sectional distributions for development variables, whose properties are quantitatively compared with the Penn World Table data set. The model generates too little dispersion in capital-output ratios and investment rates. However, it is consistent with the relative importance of investment, saving, and international capital flows for economic development.

Keywords: Economic Development, Economic Growth, Open Economy Macroeconomics, Quantitative Dynamic General Equilibrium Analysis, Incomplete Markets.

JEL Codes: E13, O11, O30, O47.

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1 Introduction

The goal of this paper is to investigate whether cross-country technological differences, which are designed to match the observed cross-country income dispersion, are also capable of accounting for development facts related to physical capital accumulation.

Specifically, this paper is interested in three sets of questions. The first set concerns cross-country dispersion in capital-output ratios: How much of this dispersion can the model account for? What fraction is due to investment-specific technological differences, and what fraction is due to neutral productivity differences? What is the contribution of international borrowing and lending? Secondly, the paper asks whether the model generates differences in investment rates and saving rates, which are as large and persistent as in the data. Finally, a third set of questions looks at the interaction between saving, investment, and cross-country capital flows. Specifically, the paper asks whether, in the model, currently richer countries have saved more, and especially invested more in the past; and whether faster-growing countries tend to save more, and especially invest more. The paper then looks at development miracles and disasters as an illustration of these regularities.

These questions are approached through a quantitative theoretical analysis of a model of an integrated world economy. The framework features a large number of small open economies, each producing a single homogeneous consumption good and a single homogeneous investment good. Financial markets are complete at the national level, but incomplete at the worldwide level. International borrowing and lending is freely allowed through trade in one-period riskless bonds, subject to a borrowing constraint. National economies are ex-ante identical, but total factor productivity (TFP) and the productivity specific to the sector producing investment goods are stochastic. Both technology shocks are purely country-specific. In part because these risks are uninsurable in world asset markets, national economies will differ ex-post. The focus is on a "long-run" scenario of time-invariant equilibrium cross-sectional distributions for the relevant variables. This scenario fits the approximate stability in world income inequality that is displayed in the Summers-Heston Penn World Table data set.

A calibrated version of this model world economy is solved numerically and its results are compared with the evidence from the Penn World Table data set. To perform this exercise, investment-specific technology levels are identified from data on the relative price of investment goods, and neutral technology levels from data on Solow residuals, appropriately adjusted for the presence of investment-specific technological differences. In this sense, by design, neutral technological differences are consistent with differences in income levels. The model is also calibrated to display a reasonable degree of cross-country capital flows.

The paper finds that technological differences of this type account for very little, from 1/5 to 1/3, of the observed dispersion in capital-output ratios, essentially all of it due to investment-specific productivity differences. This dispersion is nevertheless about 1/3 higher than what it would have been without international borrowing and lending, which reflects the contribution of the open economy framework in accounting for the basic dispersion in capital-output ratios. Since the capital share in the model is calibrated to a small value, 1/3, the model still generates about 90% of the observed dispersion in incomes, with nearly 2/3 being due to TFP differences, and 15% being due to investment-specific productivity differences.

Related to the difficulty in generating a large dispersion in capital-output ratios, investment rates and saving rates in the model are also less dispersed than in the data. They are, however, reasonably persistent, in particular they are much more persistent than income growth, and this accords well with the evidence.

Finally, overall the model is quantitatively consistent with economic development regularities pertaining to the roles of saving, investment and intertemporal capital flows. Investment rates and saving rates are strongly positively associated both with growth and with end-of-period incomes. As in the data, this is particularly true for investment rates. The data therefore suggest an important role for the access to world credit markets in economic development, which the model appears to capture particularly well.

It is worth emphasizing the fact that the development regularities in the model are generated in an open economy setting. This way, the model tries to address Lucas' (1990) question "why doesn't capital flow from rich to poor countries?" Lucas' puzzle arose because very large income differences in a neoclassical closed economy context tended to create extremely large differences in returns. Hence they tended to generate very strong incentives for cross-country movements in resources, which were hard to rationalize. A model of an integrated world economy, instead, accounts jointly for development facts and international capital flows, thus addressing Lucas' rate-of-return puzzle. The present setup is in fact entirely consistent with some degree of equilibrium dispersion in expected returns to capital, even for unconstrained economies. With incomplete international financial markets and a constraint on borrowing, countries trade in foreign assets not only to arbitrage expected return differentials on investment, but also for self-insurance reasons. Some cross-sectional dispersion in expected returns may thus be rationalized by the fact that countries with higher expected returns to domestic capital are also likely to have stronger precautionary motives for holding foreign assets.

This work relates to several other papers in the literature. It draws on Lucas (1988) in framing the problem of economic development as that of accounting systematically for the time-series and cross-sectional properties of development variables such as capital-output ratios, investment, and saving, and follows the suggestion of Klenow and Rodríguez-Clare (1997a) and earlier work by Chari et al. (1997) and Restuccia and Urrutia (2001) in employing quantitative theory. The present paper, however, offers only limited insight into the problem of economic development. It concentrates on two exogenous sources of development differences, and it is totally silent about their determinants. Attempting to understand these determinants is the fundamental question in economic development, in agreement with Prescott's (1998) call for a theory of TFP.

In trying to uncover the sources of world differences in development and growth, a distinct and fertile avenue of research has been regression analysis, typically based on the estimation of reduced-form equations using cross-sectional data (see Barro, 1991; Mankiw et al., 1992). The quantitative theory approach pursued in this paper can be regarded as complementary to the regression approach, as a way of organizing and clarifying some of the evidence around a commonly accepted model of development and growth emphasizing capital accumulation, yet it avoids endogeneity and misspecification problems, which are some of the well-known limitations of regression analysis (see Klenow and Rodríguez-Clare, 1997a,b).

Relatively few studies have explored the potential of quantitative theory to study development, Chari et al. (1997) and Restuccia and Urrutia (2001) being the first comprehensive attempts to do so. I build on their work by considering a version of the neoclassical growth model where countries are subject to idiosyncratic shocks to the variables driving cross-country differences, drawn from a common stochastic process. Restuccia and Urrutia (2001) account for the world distribution of investment rates, where countries differ only in the productivity of the capital goods sector, and the capital share is small, reflecting only payments to physical capital. In a related setting, Chari et al. (1997) account for a broader set of development facts by relying on a high capital share and intangible capital. Both papers look at a world of closed economies.

The current paper differs from the previous literature in two key dimensions. First, countries differ in terms of both investment-specific and neutral productivity. This way, one may account for a broad set of development facts while focusing on physical capital and thus retaining a conventional value for the capital share. Some of these facts turn out to be better explained quantitatively when countries differ in terms of both productivities. Moreover, this setup also allows one to isolate the individual contributions of neutral and investment-specific technological differences, which have been recognized as two potentially important sources of country differences.

Second, the current paper features an integrated world economy. One first motivation for this approach, already noted, is that it addresses Lucas' rate-of-return puzzle. In light of the puzzle, even development facts that would not necessarily call for an open economy model, should be evaluated in a framework where resources can move across countries. A successful account of country heterogeneity must not produce counterfactual implications for international differences in returns. A second motivation is that this approach provides a natural environment where to examine more distinctive open economy issues. As pointed out previously, the observed behavior of national saving differs significantly from that of investment, and thus from the predictions of the closed economy setup. Investment seems to matter more for development than saving, which points to an important role for openness. A final motivation for the open economy approach is that it amplifies the role of the capital accumulation mechanism in accounting for cross-country dispersion. The reason is that any incentive to invest domestically benefits from a leverage effect when economies are open. Essentially, in this case, foreign resources can be used along with domestic resources in investing, and national saving is not an "obstacle" to capital accumulation.

The basic model borrows from three strands of the literature. First from work on incomplete markets models, such as Clarida (1990), Huggett (1993) and Aiyagari (1994). Second, from Greenwood et al.'s (1997) model of investment-specific technical change. The present paper is, in a sense, the cross-country version of their analysis. Third, it borrows from the now extensive literature employing DSGE (Dynamic Stochastic General Equilibrium) methods, beginning with Kydland and Prescott (1982) and recently reviewed in King and Rebelo (2000).

This paper is organized as follows. Section 2 summarizes the main empirical facts on development and growth. Section 3 describes the model's assumptions and defines the equilibrium. Section 4 discusses measurement and calibration issues. Section 5 presents the results, and Section 6 concludes. Appendices A and B provide details on the empirical analysis and on the numerical algorithm, respectively.

2 Development Facts

This section presents a first qualitative description of the main relevant development facts. They pertain to variables such as world incomes, capital-output ratios, investment rates and saving rates. Section 5 complements this preliminary qualitative description with a set of summary statistics that can be easily compared with identical measures to be computed from the model's simulated data. The idea is to follow as close as possible in spirit the real business cycle literature. Hence, Section 5 provides several tables of empirical moments, which are meant to be the economic development and growth analogue of the business cycle tables of summary statistics, such as Table 1.1 in Cooley and Prescott (1995).

The data source is a panel of 125 countries from 1960 to 1985, a subset of the Penn World Table (PWT), version 5.6, described in Summers and Heston (1991). Appendix A contains the details. Most facts are well-known in the empirical growth and development literature, see for example Feldstein and Horioka (1980), Parente and Prescott (1993), Easterly et al. (1993), Barro and Sala-i-Martin (1995), Chari et al. (1997) and McGrattan and Schmitz (2000). I simply summarize these well-accepted facts, and in addition point out differences that may exist between saving and investment, as further motivation for the open economy approach pursued in this paper.

1. *World distribution of incomes.* There is a very wide range of relative incomes (defined as GDP per worker divided by the world geometric mean of GDP per worker, and henceforth denoted by RY), which has remained roughly constant through time. Both rich and poor countries, as a group, were thus able to grow at about the same rate. The shape of the empirical distribution of RY, as documented in Figure 1, changed from single-peaked with fat tails in 1960, to approximately uniform in 1985. Cross-sectional dispersion thus increased slightly from 1960 to 1985.

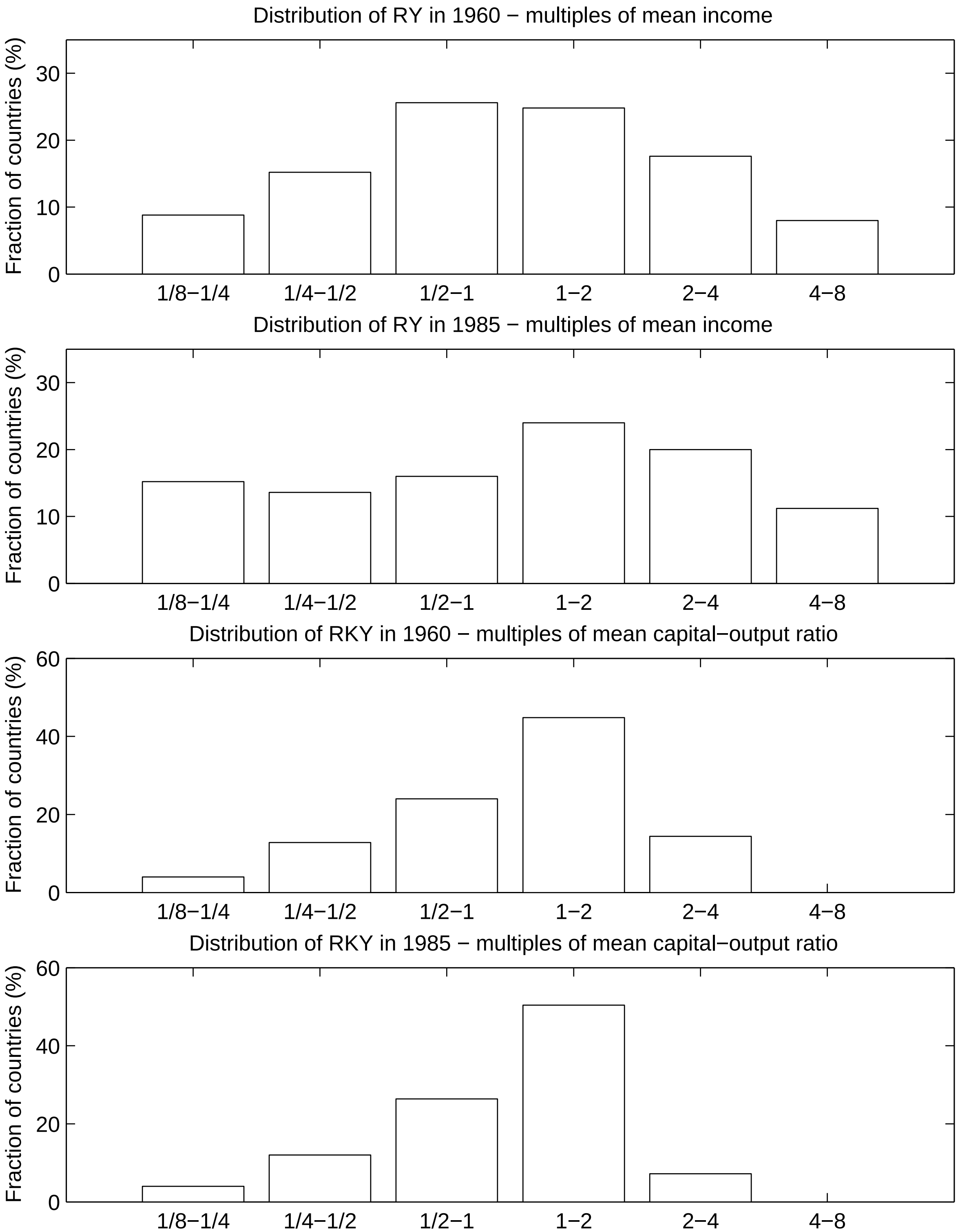

Figure 1: Incomes and Capital-Output Ratios in the Data

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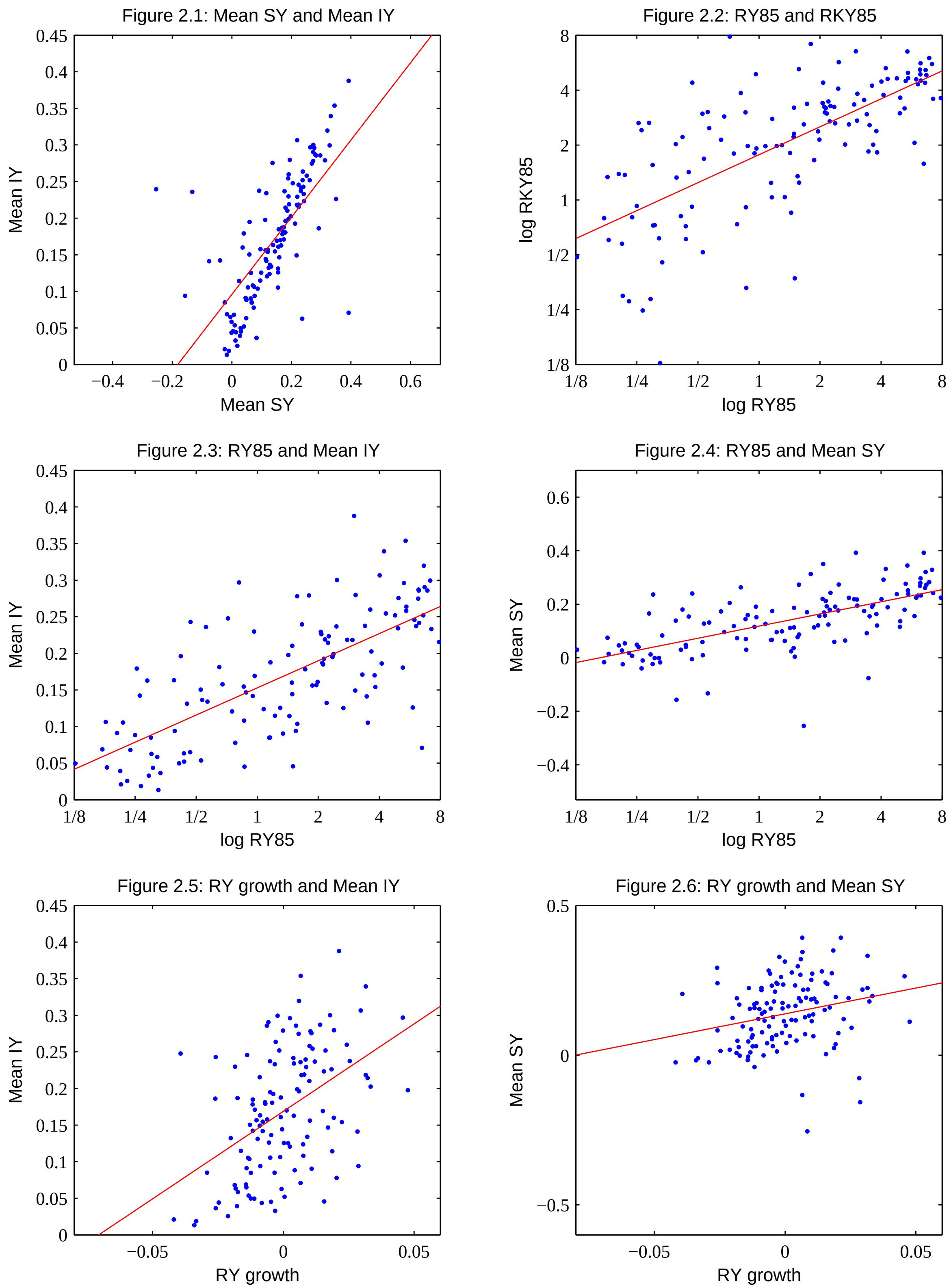

Figure 2: Scatter Plots in the Data

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2. *World distribution of capital-output ratios.* The cross-sectional dispersion in relative capital-output ratios (defined as capital-output ratios relative to the corresponding world geometric mean, and henceforth denoted by RKY) is high, although lower than that of incomes. Figure 1 also plots the world distribution of RKY in 1960 and 1985. This distribution is skewed to the left.

3. *Incomes and capital-output ratios.* Rich countries have higher capital-output ratios (Figure 2.2).

4. *Investment rates and saving rates.* There is a large cross-sectional dispersion in average investment rates (IY) and average saving rates (SY), with the dispersion in SY being significantly higher. These variables are also very persistent. Richer countries at the end of the sample period tend to have had higher SY, and especially higher IY (Figures 2.3 and 2.4). Similarly, countries with high RY growth tend to save more, but especially invest more (Figures 2.5 and 2.6). SY and IY are also highly positively correlated (Figure 2.1).

5. *World distribution of income growth rates.* Cross-sectional dispersion of growth in RY is high but RY growth is not persistent, in sharp contrast with the high persistence of IY and SY.

6. *Miracles and disasters.* The mobility within the world distribution of RY is overall low. In the growth literature it became conventional to call miracles and disasters those countries with exceptionally high and low RY growth, respectively. Development miracles differ from development disasters. Miracles invested more than the world average, but saved about the same. In contrast, disasters have both IY and SY significantly below the world average.

These facts emphasize the role of capital accumulation and openness for economic development and growth. They naturally provide the target for the model's quantitative implications. In particular, the key elements to be checked are (i) *dispersion*: how much dispersion does the model generate for RKY and RY; how much dispersion in IY, SY and RY growth; (ii) *persistence*: how much persistent are IY, SY, and RY growth; (iii) *correlations*: how well does the model capture the interaction between IY, SY, RY and RY growth; namely, how well does it capture the open economy dimension, and how well does it account for development miracles and disasters.

3 Model World Economy

The world economy is composed of a continuum of measure one of small open economies, each producing the same single consumption good and the same single investment good. All consumers have identical preferences. All firms have access to the state-of-the-art technology but, due to country-specific factors, firms in different countries may end up using different technologies.[1]

The idea is that all countries can potentially benefit from the world technological frontier, or general human knowledge, at all times. At a point in time, nevertheless, societies as a whole either make choices that prevent them from fully exploiting these benefits, or are simply unable to do so. That is, some forms of technology diffuse uniformly and instantaneously across countries, while others may diffuse differently and more slowly, depending on country-specific characteristics.

Country-specific "technological differences" refer to all the factors that affect in a broad sense societies' production possibility sets relative to the world frontier. It is a "catch-all" notion that includes not only what we may conventionally call technology, but also more generally economic policies and institutions. In this paper, cross-country technological differences are exogenous, summarized by a stochastic process. The focus of the analysis is on the effects that may result for capital accumulation, saving, and international capital flows.[2] The approach is thus one of accounting with theory. The presumption is that a deeper explanation of these technological differences would be of second-order importance in terms of accounting for the facts of Section 2.

This paper also distinguishes between differences in technology that are common to all sectors of production (differences in TFP), and differences in technology that are instead specific to the production of certain goods, namely capital. Increases in TFP affect all sectors of the economy equally and its benefits can be realized without cost. Improvements in capital-embodied technology, instead, require economies to invest and produce new capital goods in order to enjoy the increased productivity. These two sources of technological differences have been recognized in the literature as being important for studying development and growth outcomes (e.g. Greenwood et al. (1997),

[1]The emphasis on technological differences is also motivated by various growth and development accounting studies, such as Klenow and Rodríguez-Clare (1997b), and Hall and Jones (1999). A consistent finding across these studies is that technology accounts for the bulk of income differences, in both levels and growth rates, suggesting a prominent role for technology in models of development and growth.

[2]Several papers in the literature are more explicit about the determinants of these country-specific technological differences, and are thus complementary to the current approach. Some examples are Lucas (1988) for human capital differences, Eaton and Kortum (1999) for R&D and international technology diffusion, Krusell and Ríos-Rull (1996) and Parente and Prescott (2002) for endogenous technology adoption.

McGrattan and Schmitz (2000)).

3.1 Environment

The paper makes the following general assumptions about preferences, technology, and the market arrangement. To simplify notation, country subscripts are omitted.

Preferences

Each individual consumer has preferences defined over streams of consumption given by

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t u\left(C_t\right).$$

Leisure is not valued and individuals will inelastically supply their total time endowment of one unit to market activities.

Technology

The production possibilities of a national economy are given by a neoclassical production function satisfying all the usual properties

$$Y_t = F\left(A_{zt}, K_t, N_t\right),$$

where $A_{zt} = z_t X_{zt}$, and K_t and N_t are, respectively, the economy's stock of physical capital and labor input (total man-hours employed) at time t.[3] The technology parameter A_{zt} depends on a stochastic transitory component z_t which is idiosyncratic, and on a deterministic component X_{zt} common to all countries. The labor force also grows exogenously at a constant rate, which is the same for all countries.

Output can be consumed, invested or transferred to foreigners according to

$$Y_t = C_t + A_{\theta t} I_t + TB_t, \tag{1}$$

[3] Countries do not differ in terms of their stocks of human or organizational capital, i.e. there is no intangible capital. The current setup can be easily amended to have the interpretation of a one-sector model with physical and human capital, along the lines of Mankiw et al. (1992) and Chari et al. (1997). This modification amounts essentially to an increased capital share parameter, and would thus boil down simply to an issue of calibration - see Barro and Sala-i-Martin (1995), Chapter 6. See also footnote 11. The present paper chooses to concentrate on physical capital accumulation, with the added open economy dimension. Future research should consider incorporating human capital in the analysis, perhaps in a way different from the one just described.

where $A_{\theta t} I_t$ is the total expenditure in investment goods, measured in terms of consumption goods, and TB_t is the trade balance.

Physical capital depreciates at rate $\delta \in [0, 1]$ and obeys the accumulation rule

$$K_{t+1} = I_t + (1 - \delta) K_t,$$

where I_t denotes gross investment in physical capital, measured in units of capital, and is assumed to be reversible.

The marginal rate of transformation of consumption into capital goods is $A_{\theta t} = \theta_t X_{\theta t}$. It also depends on a stochastic transitory component θ_t which is country-specific, and on a deterministic component $X_{\theta t}$ common to all countries. Countries with lower θ_t are more efficient at producing investment goods, requiring a lower sacrifice in terms of consumption in order to produce one unit of capital. Under a two-sector interpretation of this framework, $A_{zt}/A_{\theta t}$ is the state of technology in the production of investment goods.[4]

The pair $(X_{zt}, X_{\theta t})$ represents the state of the world technology frontier at time t. Consistently with the development facts, these deterministic components grow exogenously at constant rates. The model is constructed to address the shape of the distributions of development variables, but not the position of the distributions themselves at a point in time. Technological progress at the worldwide level is left to be explained by endogenous growth theory.

A country's technological level may thus be indexed simply by the country-specific pair (z_t, θ_t), which by assumption follows a joint stationary first-order Markov process described by:

$$\mathbf{s}_{t+1} = \boldsymbol{\mu} + \boldsymbol{\Gamma} \mathbf{s}_t + \boldsymbol{\varepsilon}_{t+1}, \tag{2}$$

where $\mathbf{s}_t \equiv (\ln z_t, \ln \theta_t)^\top$, and $\boldsymbol{\varepsilon}_{t+1}$ is a vector of innovations. A law of large numbers is assumed to apply, and so there is no aggregate uncertainty in the world economy. The elements determining the transition, i.e. $\boldsymbol{\mu}$, $\boldsymbol{\Gamma}$ and the stochastic process for the innovations, are homogenous across

[4]According to a two-sector interpretation, consumption is $C_t = A_{zt} F(K_t^c, N_t^c) - TB_t$ and investment is $I_t = (A_{zt}/A_{\theta t}) F(K_t^i, N_t^i)$, with $K_t = K_t^c + K_t^i$ and $N_t = N_t^c + N_t^i$. Under perfect competition, with free mobility of factors across sectors, if F is CRS then it is easy to show that a single aggregate resource constraint can be written as $A_{zt} F(K_t, N_t) = C_t + \left(p_t^I / p_t^C\right) I_t + TB_t$, with $p_t^I / p_t^C = A_{\theta t}$. Investment prices are thus entirely determined by supply. The two models are isomorphic as long as investment in the one-sector model is strictly positive.

countries. Countries are ex-ante identical, and differ only ex-post, to the extent that their shock histories also differ. When concentrating on a relatively small time-window, roughly the size of the PWT data, initial conditions play an important role, as long as shocks are very persistent.

This paper thus attempts to account for country heterogeneity in the world economy without resorting to fundamental cross-country differences. This assumption, while perhaps extreme, has the advantage of imposing substantial discipline on the quantitative analysis. It is therefore a useful first step in trying to isolate the role of differences in technology in accounting for differences in development experiences.[5] Moreover, since technology will indeed be very persistent, the fact that countries are fundamentally the same in this model should not be taken too strictly. To put this into perspective, in the model it takes several thousand years for the typical very rich country to eventually become very poor.

Markets

Labor is immobile internationally, and economies interact only through intertemporal international trade. There is abundant evidence documenting a very low degree of international risk sharing, especially compared with the degree of national risk-sharing. For example, using consumption data for U.S. states, Canadian provinces and G7 countries, Atkeson and Bayoumi (1993) and more recently Crucini (1999) reported that the degree of risk sharing is higher within countries than among them. Consistent with this evidence, international financial markets are assumed to be complete at the national level, but incomplete at the worldwide level.[6] In particular, international borrowing and lending is freely allowed at the world interest rate r_{t+1}, as long as countries do not violate a borrowing constraint. The set of international financial assets is restricted to a one period riskless bond, denominated in terms of consumption.[7]

[5] Alternatively, one could imagine countries to be heterogeneous not only due to persistent idiosyncratic shocks, but also due to long-run differences. For example, one could consider differences in the parameters of the stochastic process. However, the problem could quickly become unmanageable computationally if the number of country types was sufficiently high and we insisted on an integrated open economy framework. In fact, a further motivation to study an economy with ex-ante identical countries subject to idiosyncratic shocks is that it ends up being a relatively simpler way of dealing with heterogeneity.

[6] The empirical evidence is based largely on cross-country correlations of log consumption growth. To argue that it implies this particular market structure is of course difficult - one would need to control for features such as asset market imperfections, nontradable goods, or fiscal policy effects.

[7] With reference to the two-sector interpretation of the model discussed in footnote 4, there is also an implicit assumption regarding the nontradability of either consumption or capital goods. As long as one of them is nontradable, this prevents intratemporal cross-country trades between consumption and capital goods, and international differences in the relative price of investment may subsist. The empirical evidence, however, is more favorable to the nontradability of consumption. See also Eaton and Kortum (2001) and Hsieh and Klenow (2003).

The constraint on international borrowing takes the form

$$B_{t+1} \geq \underline{B}\left(\Omega_t, z_t, \theta_t\right), \tag{3}$$

where B_{t+1} denotes the net amount of foreign assets this economy holds at the beginning of period $t+1$ and $\underline{B}\left(\Omega_t, z_t, \theta_t\right)$ denotes the minimum level allowed for these assets. The trade balance is thus $TB_t = B_{t+1} - \left(1 + r_t\right) B_t$. In anticipation of the recursive formulation of the country's problem of Section 3.3, the limit on borrowing is specified as a function of the variables that will define an economy's current state $\left(\Omega_t, z_t, \theta_t\right)$, where Ω_t denotes the total amount of resources available for domestic consumption, or a country's wealth at the start of period t:

$$\Omega_t \equiv Y_t + \left(1 - \delta\right) A_{\theta t} K_t + \left(1 + r_t\right) B_t.$$

Individuals are homogeneous within each country, even if potentially subject to purely idiosyncratic uncertainty. Since domestic financial markets are complete, one may focus on the planner's problem at the national level, equipped with the preferences and technology described above. The resulting allocation at the national level can be decentralized by various schemes (see for example Stokey and Lucas, Jr. (with Edward C. Prescott), 1989).

3.2 Model Specification

The specification of preferences and technology is standard and obeys the restrictions required for balanced growth to be possible. Momentary utility is described by a isoelastic function

$$u\left(C_t\right) = \frac{C_t^{1-\sigma}}{1-\sigma},$$

where $\sigma > 0$ is the coefficient of relative risk aversion (if $\sigma = 1$ then momentary utility is logarithmic).

The relative constancy of factor shares through time, and their approximate equality across countries (see Gollin, 2002), suggests the specification of a Cobb-Douglas production function, common to all countries, except for an idiosyncratic productivity index:

$$Y_t = A_{zt} K_t^{\alpha} N_t^{1-\alpha}, \tag{4}$$

where $0 < \alpha < 1$ is the share of physical capital. The deterministic components of technology grow at rates $X_{zt+1}/X_{zt} = \gamma_z^{1-\alpha}$ and $X_{\theta t+1}/X_{\theta t} = \gamma_\theta^{-1}$, with $\gamma_z, \gamma_\theta \geq 1$. The labor force grows at rate $\gamma_n \geq 1$. In the long-run, output and consumption in particular will grow at rate $\gamma \equiv \gamma_n \gamma_z \gamma_\theta^{\alpha/(1-\alpha)}$. Finite utility requires $\beta\gamma^{1-\sigma} < 1$, which is assumed. Variables are normalized so that $N_0 = X_{z0} = X_{\theta 0} = 1$.

In modelling the borrowing constraint, I make the simplifying assumption that

$$\underline{B}(\Omega_t, z_t, \theta_t) = -\eta\Omega_t, \tag{5}$$

so that a national economy is able to borrow only up to a multiple $\eta \geq 0$ of current resources.[8] This will provide a simple device to calibrate the model world economy to display a reasonable degree of cross-country capital flows. The constraint (5) is a tractable reduced form of a more explicit setup, capturing the basic feature that richer countries are able to borrow more.[9]

The technology shocks follow a vector autoregressive process of order one in logs. In particular, in the transition equation (2),

$$\boldsymbol{\mu} = \begin{pmatrix} \mu_z \\ \mu_\theta \end{pmatrix}, \qquad \boldsymbol{\Gamma} = \begin{pmatrix} \rho_{zz} & \rho_{z\theta} \\ \rho_{\theta z} & \rho_{\theta\theta} \end{pmatrix},$$

[8] Suppose that, due to implicit informational and commitment problems, lenders require countries to finance at least a fraction $0 \leq \kappa \leq 1$ of their current expenditures out of current domestic resources, so that $\kappa(C_t + A_{\theta t}K_{t+1}) \leq \Omega_t$. Using the resource constraint to substitute for C_t this can be written as in (5) with $\eta \equiv (1-\kappa)/\kappa \geq 0$. The parameter κ is equivalent to the margin requirement in Aiyagari and Gertler (1999). Finally, note that (3), (5) and $C_t, K_{t+1} \geq 0$ imply $\Omega_t \geq 0$.

[9] An explicit friction leading to endogenous credit constraints is lack of enforcement, in line with Kehoe and Levine (1993). Rather than exogenously assuming away insurance markets and imposing an ad-hoc borrowing constraint, as is done here, such a model would instead feature complete markets, and a set of asset-specific credit constraints would arise from the need to ensure that countries find it optimal to repay their debts rather than face exclusion from future markets.

and $\boldsymbol{\varepsilon}_t^\top \equiv (\varepsilon_{zt}, \varepsilon_{\theta t})$ is a vector of innovations which is *i.i.d.* $N(\mathbf{0}, \mathbf{\Sigma})$ with

$$\mathbf{\Sigma} = \begin{pmatrix} \sigma_z^2 & \sigma_{z\theta} \\ \sigma_{z\theta} & \sigma_\theta^2 \end{pmatrix}.$$

This specification of the stochastic process for technology shocks emphasizes parsimony, in order to simplify the computational procedure.

3.3 Equilibrium

The focus is on the economy's stationary recursive competitive equilibrium. All variables are appropriately transformed in terms of efficiency units of labor, so that they remain constant over time in the nonstochastic steady-state of the model, and are denoted with lower-case letters. We then have $\lambda_t = \Lambda_t/\gamma^t$ for $\Lambda_t = C_t, B_t$ and Ω_t, and $k_t = K_t/(\gamma\gamma_\theta)^t$. The country's state space is $S \equiv \Omega \times Z \times \Theta = \mathbb{R}_+^3$. In what follows, I denote with primes next-period variables and without primes current-period variables.

Definition *A* ***stationary recursive competitive equilibrium*** *for the world economy is a value function $v(\omega, \mathbf{s})$, a set of decision rules $c(\omega, \mathbf{s})$, $b'(\omega, \mathbf{s})$, and $k'(\omega, \mathbf{s})$, a cross-sectional distribution of countries ψ over individual states $(\omega, \mathbf{s})$, and a world interest rate r, such that:*

1. *Given r, the decision rules $c(\omega, \mathbf{s})$, $b'(\omega, \mathbf{s})$, and $k'(\omega, \mathbf{s})$ solve the country's problem and $v(\omega, \mathbf{s})$ is the corresponding value function, that is, for all $(\omega, \mathbf{s}) \in S$:*

$$v(\omega, \mathbf{s}) = \max_{c, b', k'} \left\{ u(c) + \beta\gamma^{1-\sigma} \mathrm{E}\left[v(\omega', \mathbf{s}') \middle| \mathbf{s} \right] \right\}$$

subject to

$$\begin{aligned}
\gamma\left(\gamma_\theta \theta k' + b'\right) + c &\leq \omega \\
\omega' &\equiv z'\left(k'\right)^\alpha + (1-\delta)\,\theta' k' + (1+r)\,b' \\
\gamma b' &\geq -\eta\omega \\
c &\geq 0,\ k' \geq 0 \\
\mathbf{s}' &= \boldsymbol{\mu} + \boldsymbol{\Gamma}\mathbf{s} + \boldsymbol{\varepsilon}' \\
b_0 &= 0;\ \mathbf{s}_0, k_0 > 0 \textit{ given.}
\end{aligned}$$

2. *The world capital market clears:*

$$\int_S b'(\omega, \mathbf{s})\, d\psi = 0.$$

3. *The distribution of countries is stationary and consistent with countries' behavior:*

$$\psi\left(\hat{S}\right) = \int_S P\left(x, \hat{S}\right) d\psi(x) \quad \textit{for all } \hat{S} \in \mathcal{B}_S,$$

where $P : S \times \mathcal{B}_S \rightarrow [0,1]$ *is a transition function, induced by the decision rules and the stochastic process for* $\mathbf{s}$*, and* $\mathcal{B}_S$ *is the Borel* σ*-algebra of subsets of* S*.*

3.4 Solution Method

This paper does not provide a systematic study of existence and uniqueness of equilibrium. On these questions, see Miao (2002). A closed-form solution is also not available, and I therefore resort to numerical methods.

The numerical algorithm is reasonably simple, and Appendix B contains the details. The main step of the algorithm finds the real world interest rate that clears the world capital market. For a given world interest rate, I solve an individual country's problem by value function iteration, using nonlinear continuous approximation methods. The Bellman equation holds on a finite grid for the state-space. I then use Gaussian quadrature to compute the conditional expectation on the right-hand-side of the Bellman equation, and three-dimensional cubic spline interpolation to evaluate the value function at states outside the grid. Once the equilibrium decision rules have been found, I recompute them on a much finer grid. The market outcome associated with a given world interest rate is computed by simulating the problem of an individual country for a large number of periods, where decisions at states outside the grid are found by simple linear interpolation.

4 Measurement and Calibration

I start by emphasizing that this paper is only suited to account for the facts present in the PWT time window, that is, roughly the last 30 years of data. The reason is that the steady-state of the model features a time-invariant world distribution of incomes. As reported in Section 2, this is approximately true in the PWT: the cross-sectional dispersion of incomes did remain approximately constant during this time interval.

Over a larger time horizon, however, this focus would be completely misplaced. We know this because the 21-country sample of mostly developed economies in Maddison (1991, 1994) reveals that income dispersion did increase significantly since the early 1800's (see for example Prescott (1998) and McGrattan and Schmitz (2000)). This divergence is even more dramatic if we take into account the long-run experience of today's less developed countries, as was done by Pritchett (1997). The last 30 years of data are, in this sense, a special period regarding the past evolution of the world distribution of incomes.

The long-run data therefore points to the presence of a slowly-evolving component responsible for the dynamics of the world distribution of incomes. This component might be due to the fact that industrialization diffused slowly and took-off at different dates in different countries, as suggested by Lucas (2000), with late entrants enjoying relatively faster growth.

This paper does not try to account for the long-run dynamics of the world income distribution. The view taken here is that, for the purposes of studying the PWT facts, we may abstract from whatever drives the distribution dynamics. Indeed, many development facts in the PWT, such as the highly erratic growth performance of several economies, including industrialized ones, which was documented by Easterly et al. (1993), do not appear to be linked to the slow worldwide diffusion of industrialization. This slow diffusion may nevertheless be important in accounting for the (relatively stable) shape of the world income distribution during the last three decades.

I describe next how the model is calibrated to the PWT data. The reader is referred to Appendix

A for further details. Regarding the measurement of productivity indexes, one first issue concerns the trends γ_z and γ_θ. Since the PWT does not provide a direct way to identify the latter, I assume $\gamma_\theta = 1$. Given that the goal of this paper is to account for cross-country heterogeneity in development variables, not how their trends evolve over time, this turns out to have a negligible consequence. In the quantitative analysis, all growth is therefore due to growth in X_z.[10]

A second issue is the measurement of the country-specific components θ and z. The θ's could in principle be identified directly from the relative investment price data, that is from PI^{pwt}/PC^{pwt}, where PI^{pwt} and PC^{pwt} are the PWT data for the domestic price levels of investment and consumption, respectively.[11] These relative price series, however, turn out to be fairly noisy for some countries, suggesting the existence of measurement error that could introduce a downward bias on the persistence of technological indexes. I therefore start by filtering the raw series, in order to remove the measurement error.[12] The resulting smoothed series $(PI/PC)^{sm}$ correspond to the "true" underlying panel actually used to identify θ.

In order to measure the z shocks, and consistently with the resource constraint (1), it is necessary to concentrate on output measured in consumption units, using domestic prices. In practice, this may be achieved by adjusting PPP incomes from the PWT, so that investment expenditures are measured in terms of consumption units, using domestic rather than international prices. To this end, I compute investment expenditures as $(PI/PC)^{sm}\, I^{pwt}$, where I^{pwt} is the PWT data for PPP-adjusted investment. Based upon this measure of investment, I construct a measure of adjusted output Y^a for each country-year pair given by

$$Y^a = Y^{pwt} - \left[1 - \left(\frac{PI}{PC}\right)^{sm}\right] I^{pwt}, \tag{6}$$

where Y^{pwt} corresponds to real GDP per worker in PPP terms. Notice that Y^a captures world differences in the efficiency of capital goods production, and differs from Y^{pwt} in this key respect.

[10]Alternatively, one could calibrate $\gamma_\theta = 1.03$, equal to the rate of decline of the relative price of *equipment* (not aggregate investment) based on Gordon's (1990) price series - see Greenwood et al. (1997). This would lead to a revised γ_z of 1.017. The results of Section 5, however, remain largely the same with this alternative calibration.

[11]As pointed out by Restuccia and Urrutia (2001), since the price data pertains to investment in physical capital alone, it is best suited to characterize cross-country differences in a narrow notion of capital, that excludes intangible capital.

[12]I use a simple exponential filter, with smoothing parameter equal to 0.5. The motivation to perform this filtering is merely to be on the safe side. The procedure only considers to be measurement error very wild movements in the series from one period to the next. Different filtering procedures tend to produce very similar results. An alternative approach, followed by Restuccia and Urrutia (2001), would be to average observations over some years.

This paper's results, however, will be expressed in terms output as it is measured in the PWT, i.e. they refer to:

$$y^m = y + (1-\theta)\, i, \tag{7}$$

where y and i correspond, respectively, to output and gross investment in the model, both in effective units of labor. Similarly, all results in this paper concerning saving in the model refer to $s = y^m - c$, consistently with the way I measure saving based upon the PWT data.[13]

As in most accounting exercises, such as Klenow and Rodríguez-Clare (1997b) or Hall and Jones (1999), a panel for Solow residuals is inferred from the production function (4), using data on Y^a and on the capital stock per worker. The resulting detrended observations z^{pwt} constitute measures of z.[14] For simplicity I refer to z as Solow residuals, or TFP. Rendering the series stationary is accomplished by removing a linear time trend, common to all countries. This procedure yields a point estimate for γ_z of 1.023, implying an annual growth rate of TFP of 1.5%.

Once the panel for productivity indexes is obtained, the seven parameters of stochastic process are selected so as to approximately equate seven key theoretical moments implied by (2) with the corresponding empirical moments.[15] Importantly, this ensures that the relevant properties of the distribution of productivity indexes implied by the estimated process resembles reasonably well the empirical counterparts. Table I reports the resulting parameters, and $\boldsymbol{\mu}$ is simply normalized to zero in the calibration.

Table I: VAR Parameters

ρ_{zz}	$\rho_{z\theta}$	$\rho_{\theta z}$	$\rho_{\theta\theta}$	σ_z	σ_θ	$\sigma_{z\theta}$
0.996	-0.002	0.014	0.985	0.053	0.092	0.005

The remaining benchmark parameter values, summarized in Table II, are reasonably standard. The depreciation rate is $\delta = 0.06$, and the physical capital share is $\alpha = 1/3$, consistently with

[13]In the data, government consumption is included in the consumption aggregate. Since y^m is used instead of y, some caution should be taken in interpreting the model's implications for saving. s/y^m does not capture effort in terms of current consumption. Instead, since $i = s - tb$, it simply gives an idea of the contribution of domestic resources, as opposed to foreign, to PPP-adjusted investment.

[14]Note that, since I^{pwt} varies a lot across countries while $(PI/PC)^{pwt} I^{pwt}$ does not, z^{pwt} will exhibit lower cross-sectional variation once investment-specific technological differences are accounted for.

[15]Specifically, let $\mathbf{V}_j \equiv \mathrm{E}\left(\mathbf{s}_t \mathbf{s}_{t-j}^\top\right)$, for $j = 0, 1, 2$. These theoretical moments may be computed analytically (see Hamilton (1994), pp. 265-266). The seven moments to be matched are the three in $\mathbf{V}_0$, plus the two diagonal elements of each of $\mathbf{V}_1$ and $\mathbf{V}_2$. The parameters in Table I approximately equate these moments to their empirical counterparts. Compactly, the empirical counterparts correspond to $(TI)^{-1} \sum_{i,t} (x_{it} - \bar{x}_t)(y_{it} - \bar{y}_t)$ and $((T-j)\, I)^{-1} \sum_{i,t} (x_{it} - \bar{x}_t)(x_{it-j} - \bar{x}_t)$, for $x, y \in \{z, \theta\}$, where $\bar{x}_t$ and $\bar{y}_t$ are the cross-sectional sample means, and T and I are respectively the number of years and countries in the sample.

independent evidence on income shares.[16] The parameter γ_n equals 1.019, the mean across countries of the average annual growth rate of the labor force, computed from the PWT. The coefficient of relative risk aversion is $\sigma = 1.5$, a standard choice in quantitative analysis, and in accordance with many empirical estimates.

Table II: Benchmark Calibration

β	σ	δ	γ_z	γ_θ	γ_n	α	η
0.983	1.5	0.06	1.023	1.0	1.019	1/3	0.37

Following the approach in Baxter and Crucini (1995), I select the discount factor to equate the equilibrium real world interest rate in the model to approximately 6.5%, the postwar U.S. average of the annual real return on capital (see King and Rebelo, 2000). This yields a value of $\beta = 0.983$.[17]

For the benchmark parametrization, I select η so that the model roughly matches the cross-sectional standard deviation of the trade balance to GDP in the data. The idea is to ensure a reasonable degree of cross-country capital flows. This yields a value of $\eta = 0.37$. As a sensitivity check, results are also presented for about twice this value, $\eta = 0.75$. To isolate the role of world credit markets in accounting for the development facts, I also consider the case of $\eta = 0$.

I now discuss two potential issues regarding the use of relative investment prices in the calibration. The first issue is whether relative investment prices actually reflect relative sectoral productivity differences, rather than some type of investment distortions, like taxes or tariffs. Chari et al. (1997) and Restuccia and Urrutia (2001), for example, use the same relative price data to identify investment distortions. As pointed out recently by Eaton and Kortum (2001) and Hsieh and Klenow (2003), however, most of the cross-country variation in the relative price of investment that is correlated with output is driven by the price level of consumption, not by the price level of investment. If investment distortions were the key factor behind relative price differences, we should observe the opposite. This is because prices in the PWT are sale prices, hence should reflect this type of distortions. Instead, richer countries have slightly higher investment prices. This seems to favor the technological interpretation pursued here over the investment distortions one. See Hsieh and Klenow (2003) for a more complete discussion of this point.

[16]The use of a common depreciation rate and capital share for all countries and time periods is also adopted in most development studies, such as Chari et al. (1997) and Mankiw et al. (1992). Gollin (2002) provides a justification for a common capital share, equal to about 1/3, as long as proprietors' income is accounted for as labor income.

[17]The world interest rate under incomplete markets is strictly below the one that would prevail under complete markets, which would equal 8.3%. See also Appendix B.

The second issue is related to a problem emphasized by Greenwood et al. (1997) for the temporal dimension. The issue there was that the measure of investment-specific productivity could be biased downwards if quality improvements occurring over time in capital goods were not carefully accounted for. Since this paper's focus is mainly on the cross-sectional dimension, the relevant concern here should instead be whether the correction for quality improvements is homogeneous across countries. An important element of the International Comparison Program (ICP) underlying the PWT is to ensure that the prices used to construct internationally comparable measures of real GDP and its components refer to standardized, or quality-adjusted, commodities. A potential source of bias in this paper's findings may occur if the ICP fails to carefully identify standardized capital goods for all countries in the sample relative to the U.S., say, at any given point in time. Failure to recognize quality improvements in capital goods taking place over time, but common to all countries, should simply cause γ_θ to be underestimated, in case it were inferred from the ICP price data.

5 Findings

This section is divided into two parts. Section 5.1 reports the main results, by looking at the model's implications for the statistics of interest, and by comparing them with the analogue statistics from the PWT. Section 5.2 analyzes in some detail a particular sample panel from the model. A summary evaluation of the whole exercise is left for Section 6.

5.1 Long-Run Properties

To compute the model's moments, I first approximate the unconditional long-run distributions of the variables of interest by simulation. Figure 3 depicts the long-run distributions of RY and RKY. Second, several panels of 125 countries over 26 periods were obtained, by first drawing independently 125 initial conditions from the unconditional joint distribution of states, and by then iterating forward on the model's transition function for 26 periods, starting from each initial condition. For each sample panel, I compute a set of moments that has an exact counterpart in the data. The results reported are long-run averages of the model's moments, computed over $20,000$ sample panels.

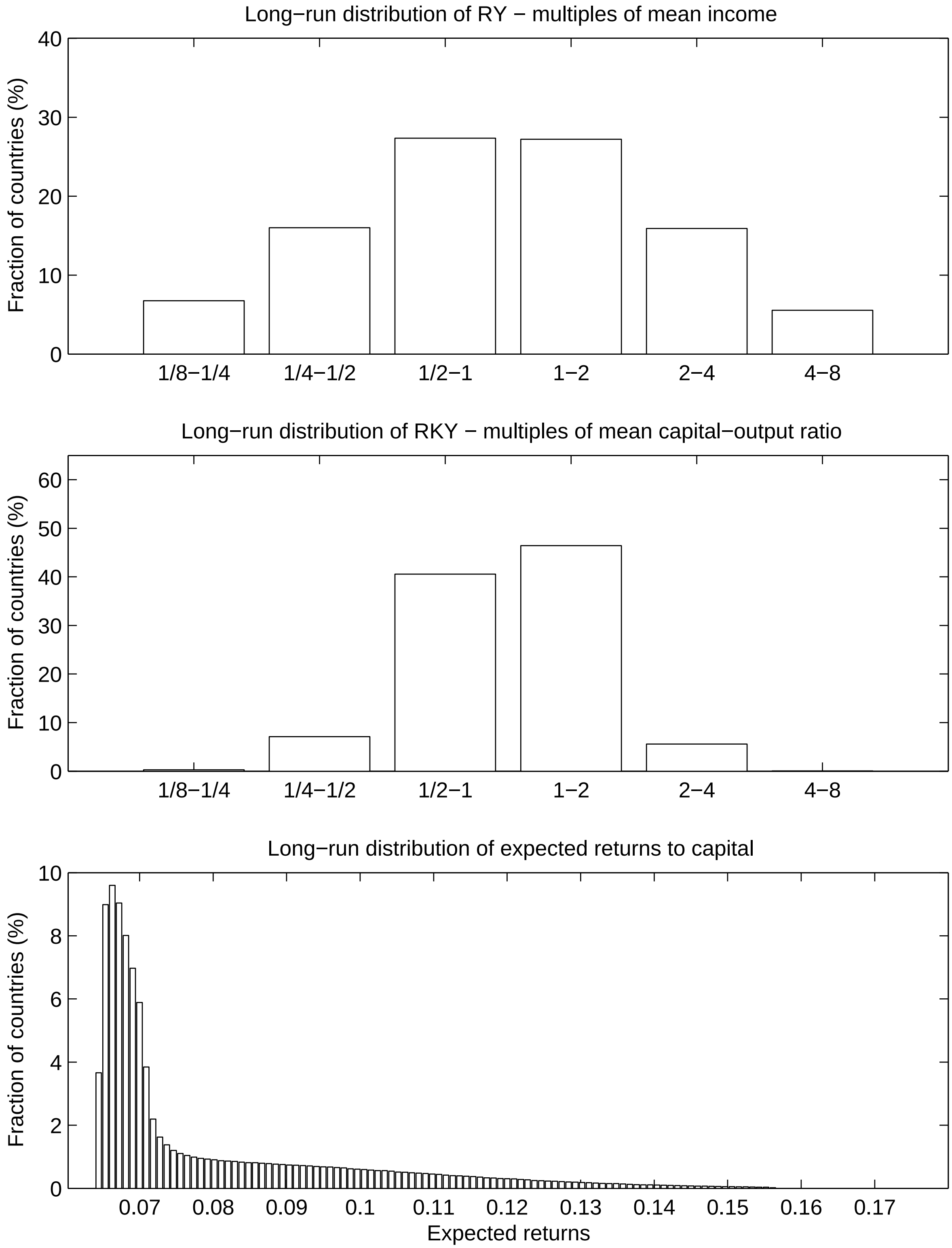

Figure 3: Long-Run Distributions

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For the benchmark calibration, the equilibrium world interest rate is 6.52% and the cross-sectional equilibrium distribution of net foreign assets implies that 31% of the countries are credit-constrained. Equivalently, countries will be constrained in world credit markets 31% of the time in the long-run. With $\eta = 0.75$, the equilibrium interest rate is 6.82%, and only 18% of the countries are credit-constrained.

Table III gives a basic assessment of the model's success in matching the cross-country dispersion in RKY. The model clearly generates too little dispersion in capital-output ratios, only accounting for about 27% of the actual dispersion for the benchmark calibration. As a result, technology shocks in fact account for just 90% of the observed variation in incomes. While TFP differences were constructed to match observed income differences, capital is an endogenous variable to the model. Since capital responds modestly to technology shocks, in the end this match is less than perfect.[18]

Table III: Cross-Country Dispersion in Incomes and Capital-Output Ratios

Variable		Dispersion	% Explained
RY	Data	1.037	-
	$\eta = 0$	0.961	0.859
	$\eta = 0.37$	0.976	0.886
	$\eta = 0.75$	0.980	0.893
RKY	Data	0.685	-
	$\eta = 0$	0.307	0.201
	$\eta = 0.37$	0.354	0.267
	$\eta = 0.75$	0.378	0.305

Notes: RY and RKY are GDP per worker and capital-output ratios relative to the world means, respectively. Dispersion is the time average of the standard deviations of the log variable. % Explained is the ratio of the model's variance to the variance in the data.

Table III also documents the significance of the integrated world economy framework for magnifying the effect of technology shocks on the dispersion in capital-output ratios. The dispersion in capital-output ratios (and similarly for incomes) does increase monotonically with η, which provides a quantitative measure of the contribution of openness in generating cross-country variation in development variables related to capital accumulation.

To understand why openness has such an effect on capital accumulation, it is useful to consider

[18]Note that output in Table III corresponds to measured output, which also helps explain why the model is able to account for most of the variation in RY without accounting for much of the variation in RKY. Measured output is in fact more dispersed (about 6% more) than output in the model, since investment is more variable when it is measured in investment units rather than in consumption units.

the first-order conditions for an unconstrained economy in the model. After some simple rearrangements, these conditions imply the following relationship between the expected returns to domestic capital and the world interest rate:

$$\mathrm{E}_{\mathbf{s}}\left[\frac{\alpha y'/k'+\theta'(1-\delta)}{\gamma_{\theta}\theta}-1\right]=r+\pi(\omega,\mathbf{s}), \tag{8}$$

where

$$\pi(\omega,\mathbf{s})\equiv-\operatorname{cov}_{\mathbf{s}}\left\{\beta\gamma^{-\sigma}(1+r)\frac{u'(c')}{u'(c)},\frac{\alpha y'/k'+\theta'(1-\delta)}{\gamma_{\theta}\theta}\right\}$$

is the risk premium. Eq. (8) says that the expected marginal return on capital, measured in consumption units, must equal the world interest rate, which is the return on foreign assets measured in terms of consumption, plus a risk premium. To help develop some intuition, take $\pi(\omega,\mathbf{s})$ to be a constant in the discussion that follows.

Eq. (8) gives a first indication of the importance of θ shocks in generating differences in capital-output ratios: θ introduces a wedge between (ex-ante) capital-output ratios and the world interest rate. What this means is that, even if the returns to capital are roughly equalized in terms of consumption units among unconstrained economies, countries do differ in how efficient they are in transforming consumption goods into capital goods. Hence, they may end up with very different capital-output ratios.

Now notice how this fact interacts with openness. For an unconstrained economy, it is clear from (8) that its capital-output ratio is going to be very responsive to θ. As a consequence, among unconstrained economies, the dispersion in capital-output ratios tends to be strongly associated with the dispersion in the θ's. Compare this scenario with the one studied by Chari et al. (1997) and Restuccia and Urrutia (2001), where all economies are closed ($\eta=0$). In this case, since there is no world interest rate pinning-down the expected returns to capital across countries, capital-output ratios become less responsive to θ. Intuitively, without access to international credit, increases in domestic investment must be matched by corresponding increases in saving and thus, in a sense, saving works as a constraint on the extent to which investment may respond to differences in θ. More specifically, in a closed economy there is an endogenous response of the domestic real interest rate, which works to discourage investment.

What Table III also reveals, however, is that this effect is relatively small. Even when relaxing the borrowing constraint all the way from $\eta = 0$ to $\eta = 0.75$, the explanatory power of the model for the disparity in RKY only increases from 20% to 30%. This means that the cross-country variance in RKY is about 34% larger in the benchmark calibration, but even then it is too small an effect to matter a lot for incomes. Since the capital share in the model is only 1/3, it translates into a much lower increase in the dispersion of RY. An open question remains as to which other exogenous factors, or which other additional endogenous model mechanisms, would be capable of generating capital-output differences as large as those observed in the data.

Turning now to Table IV, we notice a related implication, that also the cross-sectional dispersion of SY, and especially that of IY, is not as high as in the data: for IY, the coefficient of variation in the model is 62% of the coefficient of variation in the data, and for SY this number is 72%.[19] In terms of growth rates, the cross-sectional variation in the model is 88% of the variation in the data.

Table IV: Investment and Saving Rates, and Income Growth

Variable		Min 5%	Max 5%	Dispersion	Persistence
Mean IY	Data	0.025	0.335	0.506	0.836
	$\eta = 0$	0.108	0.450	0.341	0.770
	$\eta = 0.37$	0.118	0.467	0.312	0.487
	$\eta = 0.75$	0.111	0.457	0.317	0.385
Mean SY	Data	-0.114	0.357	0.825	0.824
	$\eta = 0$	0.108	0.450	0.341	0.770
	$\eta = 0.37$	-0.132	0.477	0.584	0.797
	$\eta = 0.75$	-0.227	0.485	0.697	0.827
RY growth	Data	-0.034	0.037	0.017	0.158
	$\eta = 0$	-0.026	0.028	0.013	0.201
	$\eta = 0.37$	-0.032	0.031	0.015	0.069
	$\eta = 0.75$	-0.032	0.033	0.016	0.050

Notes: Mean IY and Mean SY are the time averages of investment and saving rates. RY growth is the average annual growth rate of RY. The first two columns are the averages of the lowest and the highest 5% values, respectively. Dispersion is the coefficient of variation for Mean IY and Mean SY, and the standard deviation for RY growth. Persistence is the correlation of the average for the first 13 periods with the average for the last 13 periods.

Consistently with the data, the model features a higher dispersion of saving rates than invest-

[19]The result for Mean IY is thus similar to an analogous result in Restuccia and Urrutia (2001), although they do not look at average investment rates. It should be pointed out, however, that the ratio of the coefficients of variation does not have a variance decomposition interpretation, in contrast to the last column of Table III. Hence, while one cannot claim that, with independence, technology shocks alone account for 62% of the dispersion in investment rates while the remaining 38% are due to other factors, the ratio of coefficients of variation facilitates the comparison with previous work.

ment rates. In fact, according to the model, as the world economy becomes more open (higher η), the dispersion in SY tends to become larger than the dispersion in IY. In fact, the latter is not at all sensitive to η. This suggests that countries may save at widely different rates but, because they may rely on foreign resources in investing, the dispersion in investment rates tends to remain anchored mostly to the dispersion in investment-specific productivities.

A further point about Table IV is that investment rates are less persistent in the model than in the data, the same being true for RY growth. Still, the model is consistent with RY growth being much less persistent than IY or SY. The main reason for this lack of persistence of IY is the absence in the model of any frictions to international capital flows. Since countries produce a single, perfect substitutable consumption good, investment (and also the trade balance) is highly responsive to shocks when economies are unconstrained. As it is standard in the literature, a more realistic behavior for these variables could be achieved by introducing frictions, either on capital accumulation (as in, say, Baxter and Crucini, 1995) or on international trade itself (as in Backus et al., 1992).[20]

Table V gives an idea of the interaction between the various variables related to capital accumulation. First, the saving-investment correlation is positive and high, in fact slightly higher than in the data. This is the case despite a fairly generous borrowing limit. The model thus speaks against inferring low capital mobility from this high correlation, as was done in Feldstein and Horioka (1980).[21] Second, richer countries at the end of the sample tend to have had high saving rates on average, and especially high investment rates, just as in the data. Similarly, countries with higher RY growth usually have high average saving rates, and especially high average investment rates.[22] Therefore, faster-growing countries in the model tend to be borrowers in world credit markets, and richer countries tend to have been borrowers in the past. The model thus highlights a link between openness and economic development which appears to be quite consistent with observation.

Table VI looks at the development experiences of miracles and disasters. These are the countries

[20]In the present case, the implementation of such features pose a computational difficulty: the composition of wealth would become a state variable if, as it is standard, adjustment costs are specified as a function of the change in assets.

[21]There are several papers in the literature making a similar point in different contexts. Baxter and Crucini (1993) is an example among many others.

[22]In the PWT data, the SY-RYgrowth and IY-RYgrowth correlations are significantly different from each other at the 1% significance level. The SY-RY85 and IY-RY85 correlations are only significantly different at the 10% significance level. These results are based upon the Z-test for the equality of two correlation coefficients.

Table V: Correlations Between Income, Investment and Saving

		Mean IY	Mean SY	RY 85
Mean SY	Data	0.703	1.0	
	$\eta = 0$	1.000	1.0	
	$\eta = 0.37$	0.768	1.0	
	$\eta = 0.75$	0.679	1.0	
RY 85	Data	0.706	0.647	1.0
	$\eta = 0$	0.511	0.511	1.0
	$\eta = 0.37$	0.479	0.439	1.0
	$\eta = 0.75$	0.439	0.401	1.0
RY growth	Data	0.464	0.250	0.399
	$\eta = 0$	0.543	0.543	0.171
	$\eta = 0.37$	0.591	0.418	0.190
	$\eta = 0.75$	0.623	0.364	0.197

Notes: Mean IY and Mean SY are the time averages of investment and saving rates. RY growth is the average annual growth rate of RY. RY 85 is log RY in 1985 in the data and end-of-period log RY in the model's sample panels.

whose past growth experience is markedly different from the world norm. Both in the data and in the model, they are defined to be the 10 countries with the highest and the lowest growth rates of RY, respectively.

The model mimics reasonably well these experiences. The main discrepancy is that the model predicts miracles to be countries with exceptionally high saving rates, whereas in the data they are similar to the world average. The reason is simply that, in the model, miracles tend to be credit-constrained initially, which leads SY to track IY closely. Still, miracles in the model are borrowers in international markets, a feature which is consistent with the data.

5.1.1 Relative Contributions of Technology

This section tries to better understand the results of Section 5.1, by isolating the individual contributions of neutral and investment-specific technology differences. In a sense, this may be viewed as the cross-country analogue of the exercise in Greenwood et al. (1997), who perform a similar decomposition for the long-run growth in US output. The results are summarized in Tables VII through IX. Overall, they show that both shocks are in fact important in accounting for the development facts related to capital accumulation.

Table VII provides a decomposition of the cross-sectional dispersion in incomes and capital-output ratios. The line labelled "Total" simply reproduces Table III for the benchmark case of

Table VI: Miracles and Disasters

Group Averages				
Group		RY growth	Mean IY	Mean SY
Miracles	Data	3.34%	0.225	0.139
	$\eta = 0$	2.51%	0.343	0.343
	$\eta = 0.37$	2.74%	0.362	0.334
	$\eta = 0.75$	2.89%	0.368	0.337
Disasters	Data	-3.01%	0.092	0.078
	$\eta = 0$	-2.34%	0.172	0.172
	$\eta = 0.37$	-2.81%	0.174	0.109
	$\eta = 0.75$	-2.86%	0.172	0.108
World	Data	0.0%	0.169	0.138
	$\eta = 0$	0.0%	0.250	0.250
	$\eta = 0.37$	0.0%	0.270	0.241
	$\eta = 0.75$	0.0%	0.269	0.240

Notes: Mean IY and Mean SY are the time averages of investment and saving rates. RY growth is the average annual growth rate of RY. Miracles and Disasters are the 10 countries with the highest and the lowest growth rates of RY, respectively. Group averages are arithmetic averages.

$\eta = 0.37$. The line labelled z pertains to a version of the model where θ is set equal to its unconditional mean, and similarly for the line labelled θ. In both these versions, the decision rules and the world interest rate correspond to the solution of the model when both shocks are allowed to operate.

Table VII: Cross-Country Dispersion in Incomes and Capital-Output Ratios: Relative Contributions

Variable	Source	Dispersion	% Explained	% Contribution
RY	Total	0.976	0.886	1.0
	z	0.774	0.557	0.627
	θ	0.375	0.131	0.148
RKY	Total	0.354	0.267	1.0
	z	0.070	0.010	0.039
	θ	0.360	0.276	1.034

Notes: RY and RKY are GDP per worker and capital-output ratios relative to the world means, respectively. Dispersion is the time average of the standard deviation of the log variable. % Explained is the ratio of the model's variance to the variance in the data. % Contribution is the ratio of the model version's variance to the model's total variance (the covariance term is ignored).

Neutral technology differences contribute for most of the dispersion incomes, nearly 2/3, whereas investment-specific productivity differences account for essentially all of the dispersion in capital-output ratios.

This can be easily understood from (8). Neutral shocks may generate large and persistent differences in incomes, mainly because they impact directly on output, but their effect on capital-output ratios is very limited quantitatively. In fact, any such effect tends to arise mostly on impact, while the capital stock is fixed.[23] Instead, as already explained in Section 5.1, investment-specific shocks tend to produce large and persistent differences in capital-output ratios. However, since θ shocks operate through the capital accumulation channel, and since the capital share in the model is small, they do not translate into very significant income differences.

Both types of shock thus seem necessary to generate quantitatively reasonable differences in incomes. According to Table VII, if countries differed only in terms of TFP, then the model would only account for a little over one half of the observed dispersion. The full model is able to account for about 90% of the dispersion in incomes due to the endogenous response of capital-output ratios, particularly to variations in investment-specific productivities.

Table VIII: Investment and Saving Rates, and Income Growth: Relative Contributions

Variable	Source	Min 5%	Max 5%	Dispersion	Persistence
Mean IY	Total	0.118	0.467	0.312	0.487
	z	0.180	0.276	0.100	0.039
	θ	0.109	0.454	0.257	0.205
Mean SY	Total	-0.132	0.477	0.584	0.797
	z	0.198	0.274	0.072	0.470
	θ	-0.074	0.467	0.538	0.673
RY growth	Total	-0.032	0.031	0.015	0.069
	z	-0.029	0.029	0.014	0.038
	θ	-0.030	0.030	0.014	-0.031

Notes: Mean IY and Mean SY are the time averages of investment and saving rates. RY growth is the average annual growth rate of RY. The first two columns are the averages of the lowest and the highest 5% values, respectively. Dispersion is the coefficient of variation for Mean IY and Mean SY, and the standard deviation for RY growth. Persistence is the correlation of the average for the first 13 periods with the average for the last 13 periods.

In Table VIII we see that the cross-country dispersion in both θ and z play an important role in accounting for the differences in investment rates, saving rates, and income growth. Consistently with the previous discussion, differences in θ alone are key in generating the model's dispersion and persistence in both investment rates and saving rates. However, there is also an independent contribution from TFP, as well as interaction effects. The model with the two shocks is more successful in accounting for the facts.

The message from Table IX is similar. In this table, notice in particular the importance of

[23]This is related to a well-known implication of the standard neoclassical growth model with Cobb-Douglas production, that capital-output ratios are unrelated to TFP differences along a balanced-growth path.

Table IX: Correlations Between Income, Investment and Saving: Relative Contributions

	Source	Mean IY	Mean SY	RY 85
Mean SY	Total	0.768	1.0	
	z	0.686	1.0	
	θ	0.696	1.0	
RY 85	Total	0.479	0.439	1.0
	z	0.444	0.379	1.0
	θ	0.784	0.821	1.0
RY growth	Total	0.591	0.418	0.190
	z	0.936	0.625	0.229
	θ	0.564	0.145	0.460

Notes: Mean IY and Mean SY are the time averages of investment and saving rates. RY growth is the average annual growth rate of RY. RY 85 is log RY in 1985 in the data and end-of-period log RY in the model's sample panels.

having both shocks in generating the right correlation between saving/investment rates and the level/growth rate of incomes. Neutral shocks tend to generate high saving-growth and investment-growth correlations, but not such high correlations with income levels. The opposite is true for investment-specific shocks.

The intuition for this result follows from previous discussion. The average level of the investment and saving rates for a given country tends to be largely independent from the average level z. In contrast, the level of z is a key determinant of the level of output. This explains the relatively low correlation with income levels. In turn, the relatively high correlation with income growth is explained by the fact that, whenever a country experiences high growth in the model, this is due to a positive z shock. In response to this shock, both investment and saving, but particularly the former, tend to increase significantly. Even though these increases are concentrated in the periods immediately after the shock, they are sufficient to generate the high correlations.

Something different occurs with θ shocks. These shocks are very important in determining the average level of saving and investment, and also (to a less extent) income levels. However, shocks to θ are not able to generate significant output growth. The message is once again that a combination of both shocks appears to offer a better overall account of the facts.

5.1.2 International Dispersion in Returns to Capital

As pointed out previously, the open economy setting ensures that the heterogeneity generated by the model is not at odds with the incentives to move resources across countries. Basically, this guarantees that the wide range of development experiences summarized in Tables III through VI does not rely upon significant barriers to international capital mobility.

To judge the importance of this feature, and in the spirit of Lucas (1990) and King and Rebelo (1993), one may ask whether the benchmark economy generates a plausible degree of international dispersion in expected returns to domestic capital. To help answer this question, the bottom of Figure 3 plots the long-run distribution of expected returns in the model, restricted to the interpercentile range containing 99% of the observations.[24]

This distribution is non-degenerate, even when restricted to unconstrained economies. In other words, the current setup features what might be called "arbitrage gaps" in expected returns to capital. These gaps are perfectly consistent with equilibrium, in the sense that unconstrained economies have the ability to exploit them, but find it optimal not to do so. The reason is that countries have two motives to trade in foreign assets. The first motive is standard: economies simply wish to exploit international differences in the return to investment. The second motive has to do with the fact that foreign assets provide self-insurance. The first motive implies that, if the expected return on domestic capital is higher than the world interest rate, countries will run down their stock of foreign assets, and eventually borrow from abroad, in order to finance domestic investment. The second self-insurance motive implies in turn that, if the arbitrage gap is not sufficiently large, it may actually be advantageous for a country to renounce it. The benefit is that this allows the economy to keep a buffer stock of foreign assets, as a precaution against the event of becoming credit-constrained in the future. As a result, expected returns on domestic capital are not necessarily equated across countries, even among unconstrained ones.

With respect to the long-run distribution of expected returns in Figure 3, I consider the ratio of the upper to the lower bound of expected returns. This largest factor difference in the model turns out to be 2.4, and it is 1.2 within the set of unconstrained economies. This means that, even including economies that are up against the borrowing constraint, these are reasonably small numbers. For comparison purposes, if the world economy were a collection of closed economies (the case of $\eta = 0$), then the largest factor differences in expected returns increase to about 9, that is, by a factor of nearly 4. Clearly, in order to rationalize such large dispersion in rates of return,

[24] Expected returns, measured in consumption units, correspond to the left-hand-side of (8). They can be computed without resorting to approximation methods, using the fact that $\mathbf{s}'$ is conditionally lognormal.

substantial impediments to capital flows would have to be invoked.

5.2 Sample Panel

This section looks at the model's implications from a slightly different perspective, by analyzing in some detail a particular sample panel from the model. The interest in this panel stems from the way it is constructed, such that both the initial 1960 conditions and the subsequent histories of shocks are matched to the observations from the PWT.

Specifically, the panel for shocks corresponds exactly to the PWT's, meaning that it is the one inferred from the data using the methods described in Section 4. To obtain initial conditions for capital and net foreign assets, and hence wealth, the model is required to match some selected observations in the 1960 PWT cross-section. In particular, given the PWT's initial distribution of shocks (z_0, θ_0), the model's decision rules are used to compute the pairs k_0, k_1 and b_0, b_1 such that the initial distributions of RKY and IY match exactly the corresponding PWT distributions in 1960.[25] The remaining sequences of capital stocks and net foreign asset levels are obtained by iterating forward on the decision rules, and by using the actual panel for productivity indexes as the shock realizations. At all times, countries expect shocks to be drawn from the stochastic process (2). The model's parameters are those of the benchmark calibration.

Figure 4 displays the cross-sectional distributions of RY and RKY for the starting and ending periods of the simulation, and it is the sample panel's counterpart to Figure 1. For convenience, they are plotted together with the PWT's. In turn, Figure 5 is the counterpart to Figure 2.

These figures are consistent with much of the discussion in Section 5.1. Figure 4 illustrates the difficulty the model has in generating the observed dispersion in capital-output ratios, particularly the left tail of the distribution of RKY. This figure reveals that, for the model to be consistent with the initial distribution of RKY, which here is true at $t = 1$ by construction, the initial distribution of RY needs to be more skewed to the right than in the data, implying a shift in mass towards very poor countries. From $t = 1$ onwards, the distributions of RY and RKY begin converging towards the long-run distributions shown at the top of Figure 3. At the end of the sample ($t = 26$),

[25] I end up with a panel of only 120 countries since Namibia, Peru, Suriname, Gabon and Somalia had to be discarded. Their own data together with the model's decision rules implied negative measured output in the model for 1960. This is the reason why, in Figure 4, the initial distribution of IY in the model does not appear to match perfectly the corresponding distribution in the PWT.

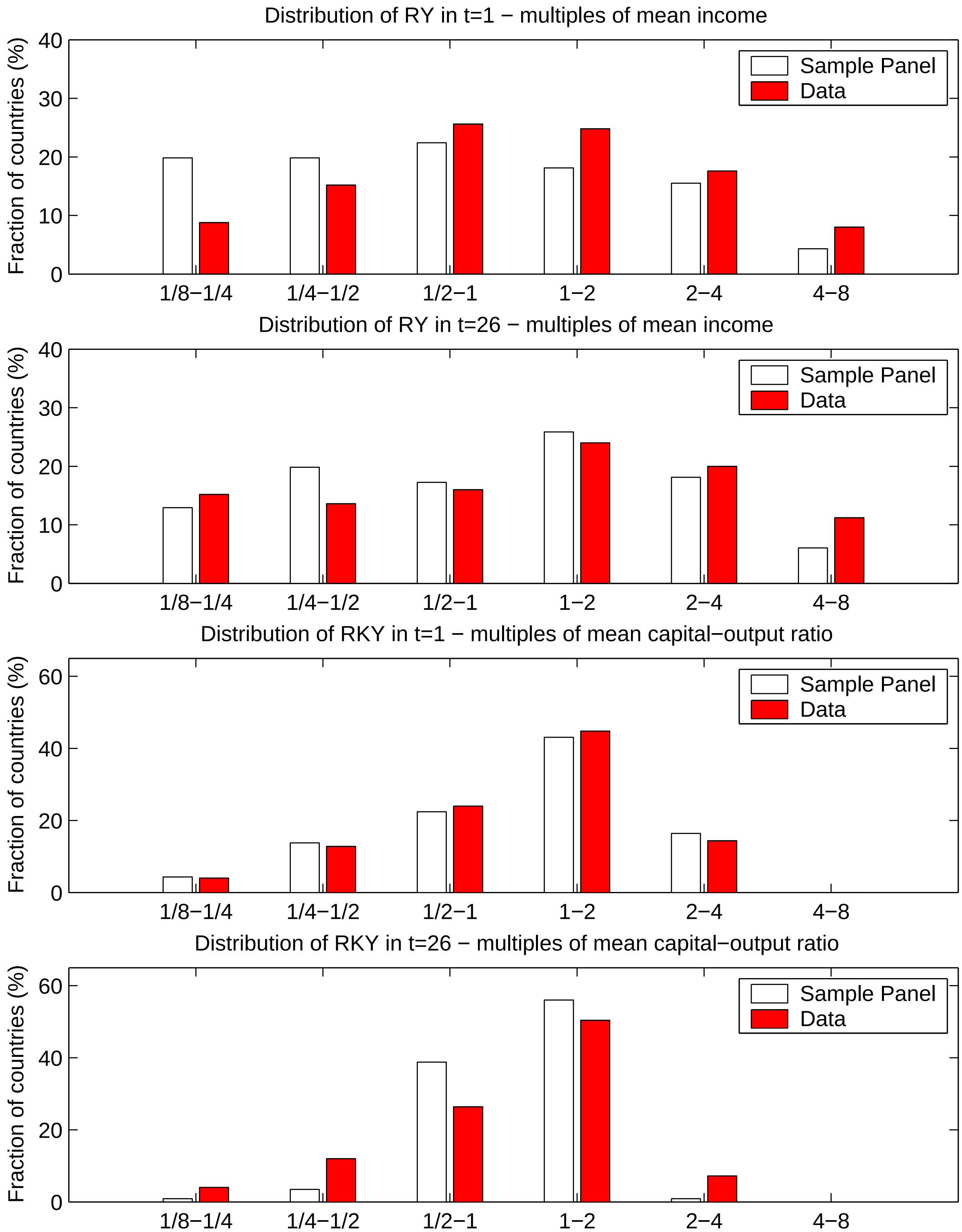

Figure 4: Incomes and Capital-Output Ratios in the Model

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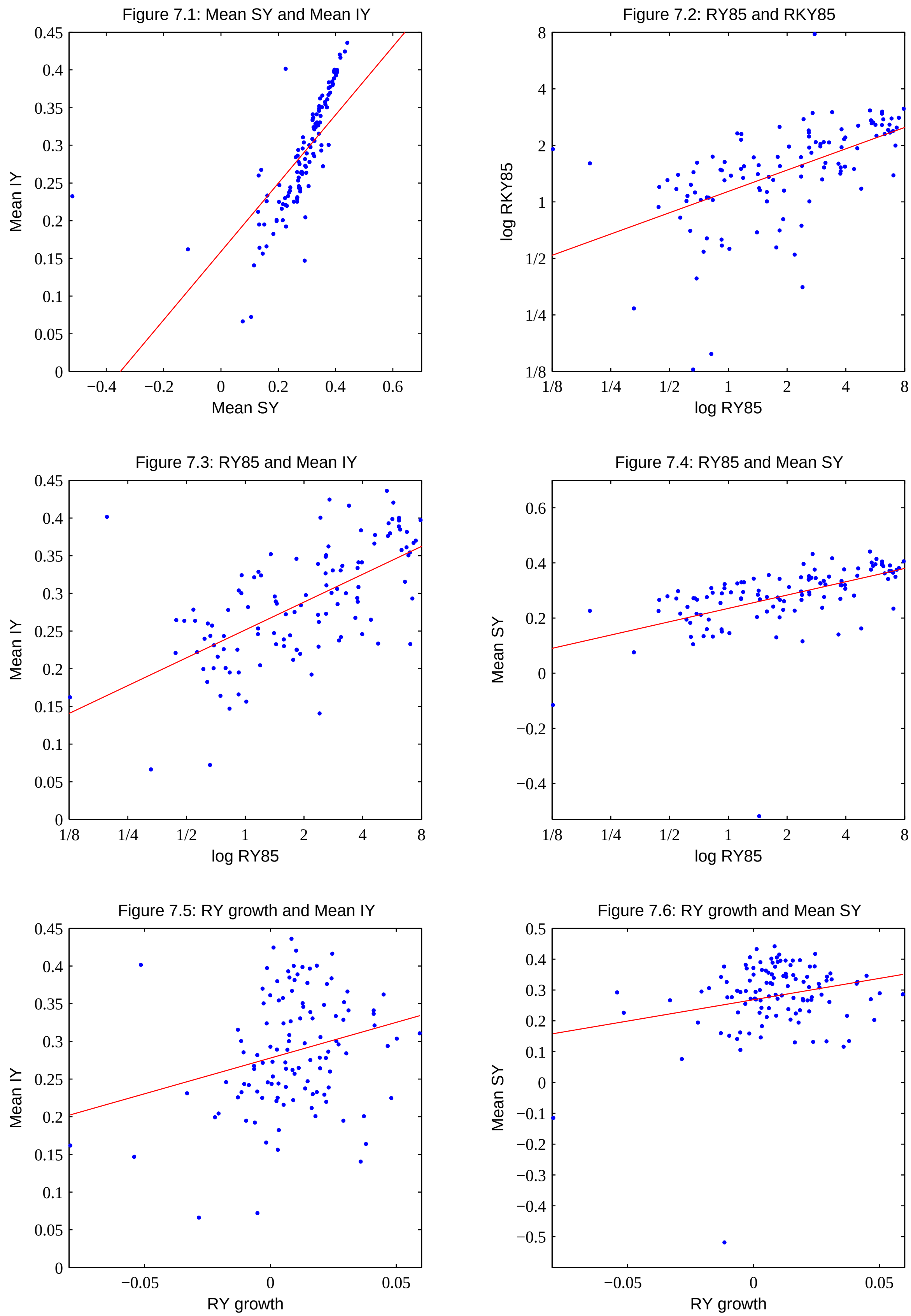

Figure 5: Scatter Plots in the Model

the distribution of incomes in the model thus looks much similar to the actual one. Instead, and consistently with the findings of Table III, the distribution of capital-output ratios is distinctly less dispersed than in the data.

Figure 5 gives a clear indication of how well the model captures the interaction between the various development variables related to capital accumulation. Beyond the simple correlations of Table V, the overall shape of the different clouds of points generated by the model looks remarkably similar to those in Figure 2. Of particular relevance is the fact that investment rates are clearly more strongly associated with economic development and growth (Figures 5.3 and 5.5) than saving rates (Figures 5.4 and 5.6).

This is consistent with an important role for access to international credit markets in accounting for economic development. The PWT countries that grew faster and those that are richer have accumulated more capital, and for that they have relied on foreign resources. The model predicts precisely this type of behavior, given the actual shock realizations we infer these countries have been subject to in the data.

6 Conclusion and Remarks

This paper asks whether technological differences across countries, which are designed to be consistent with cross-country income differences, are also consistent with development facts related to capital accumulation. This problem is addressed through a quantitative dynamic general equilibrium analysis of a model of an integrated world economy. The model generates a wide range of implications, which are compared with the data from the Penn World Tables. Since national economies interact in world markets, the model accounts jointly for development facts and international capital flows. This means that the model's implications for world heterogeneity are not necessarily in conflict with the incentives to move resources across countries.

The model's overall success is mixed. On the negative side, it accounts for too little of the observed dispersion in capital-output ratios. Similarly, the dispersion in investment rates is lower in the model than in the data. The bulk of the model's dispersion in these variables comes from differences in investment-specific productivity. While the open economy approach is shown to amplify the effect these differences have on capital-output ratios and investment rates, they turn out to be relatively limited from a quantitative standpoint. Further research is needed in order to account for the large cross-sectional variation in capital-output ratios. One potentially fruitful direction is to recognize that the notion of the relative price of investment most relevant for the identification of investment-specific technology indexes is a broad *effective* notion, which includes all investment costs, and not just the market price of capital. Most likely, observed world differences in the relative price of investment goods underestimate differences in the true effective price. Better theory, specifying additional and possibly implicit investment costs capable of amplifying observed international differences in investment prices, could potentially improve the predictions of model in this dimension.

On the positive side, the model provides a reasonably good description of the interplay between the various development variables related to capital accumulation. In particular, faster-growing countries save more, and especially invest more. Similarly, currently richer countries have saved more, and especially invested more in the past. Cross-country saving-investment correlations are therefore high, even for economies with unrestricted access to international capital markets. The model thus appears to describe reasonably well the interaction between openness and capital accumulation, and its relation to economic development.

The analysis in this paper has two important limitations. First, it says very little about what drives cross-country income differences. While this is without doubt the central question in economic development, in the model the bulk of these differences comes from TFP, the residual. This paper therefore merely underscores the importance of understanding why TFP differs so much across countries. A second limitation of the analysis is that it says nothing about economic development beyond the time window of the Penn World Tables. The world income distribution has in fact been changing significantly over the very long-run, and understanding why this is the case requires a model different from the one in this paper, where the world income distribution exhibits some dynamics.

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A Data

The data set is Summers and Heston's (1991) Penn World Tables (PWT), version 5.6. I concentrate on a panel of 125 countries from 1960 to 1985, the largest panel with complete data for real GDP per worker and real gross investment, and the same used in Chari et al. (1997) and Restuccia and Urrutia (2001). Following Parente and Prescott (1993) and Chari et al. (1997) I focus on relative world incomes, defined by real GDP per worker (variable RGDPW in PWT) relative to the world geometric average.

Gross investment, valued at 1985 international prices, is computed as the product of the PWT variables I, POP and RGDPCH. Capital stocks are constructed by the perpetual inventory method. Following Klenow and Rodríguez-Clare (1997b) and Hall and Jones (1999) I assume countries were in their nonstochastic steady-states until 1960. The initial capital stock per worker for a given country is:[26]

$$K_{60} = \frac{I/Y}{\gamma_y \gamma_n - 1 + \delta} Y_{60},$$

where I/Y and γ_n are the averages over the 1960-70 period of, respectively, the country's gross investment rate and growth rate of the labor force, and Y_{60} is the initial real GDP per worker. The depreciation rate δ is 0.06 and $\gamma_y = 1.023$, the annual growth rate of the geometric world average of output per worker.

I now discuss in more detail the rationale for measuring investment expenditures as $\left(PI^{pwt}/PC^{pwt}\right) I^{pwt}$. With respect to the resource constraint (1), the model's measures of real consumption and real gross investment should have a correspondence to C^{pwt} and I^{pwt}. To ensure cross-country comparability, the PWT reports expenditures computed using a common set of base-year international prices for each specific good category. PWT expenditures in real terms can thus be denoted by $C_{it}^{pwt} = \pi_b^C C_{it}$ and $I_{it}^{pwt} = \pi_b^I I_{it}$, for country $i = 1, \ldots, N$ and year $t = 1, \ldots, T$, where π_b^C and π_b^I are the relevant international prices for base year b, which multiply the appropriate physical quantities.[27] Nevertheless, according to (1), investment expenditures should be measured in terms of consumption units, using current domestic prices. In addition, they ought to be denominated in base-year international consumption prices, to preserve comparability with the PWT. A measure of output in the PWT comparable to the one in the model should thus be based upon the following measure of investment:

$$\pi_b^C \frac{p_{it}^I}{p_{it}^C} I_{it} = \frac{\pi_t^I/\pi_t^C}{\pi_b^I/\pi_b^C} \frac{p_{it}^I/p_{it}^C}{\pi_t^I/\pi_t^C} \pi_b^I I_{it} = \frac{\pi_t^I/\pi_t^C}{\pi_b^I/\pi_b^C} \frac{PI_{it}^{pwt}}{PC_{it}^{pwt}} I_{it}^{pwt},$$

where p_{it}^I/p_{it}^C is the domestic relative price of investment, and PI_{it}^{pwt} and PC_{it}^{pwt} are the PWT data

[26] The need to compute initial capital stocks indirectly arises from the lack of direct estimates in the PWT for a large number of countries. This particular procedure has the advantage of not imposing end-of-period capital stocks to be on a balanced-growth path. The results are not very sensitive to the procedure used to compute the initial capital stock (see Klenow and Rodríguez-Clare, 1997b; McGrattan and Schmitz, 2000).

[27] In reality, the PWT computes expenditures in good j by multiplying world prices relative to the U.S. price ($\overline{\pi}_{it}^j \equiv \pi_{it}^j/p_{ust}^j$) and quantities valued at U.S. dollars (notional quantities, or $Q_{it}^j \equiv p_{ust}^j q_{it}^j$). For details see Kravis et al. (1982).

for the domestic price levels of investment and consumption, respectively. These are reported as Purchasing Power Parities, and so $PI_{it}^{pwt}/PC_{it}^{pwt}$ in fact corresponds to the ratio of the domestic relative price of investment to its international counterpart. In line with the two-sector interpretation of the current setup, it is legitimate to identify A_θ with p^I/p^C. Further, if we identify X_θ with π^I/π^C, we have $\left(\pi_t^I/\pi_t^C\right)/\left(\pi_b^I/\pi_b^C\right) = 1$, given that $\gamma_\theta = 1$.

The panel for the Solow residuals, A_{zit}, is constructed from the neoclassical production function, $Y_{it}^a = A_{zit}K_{it}^\alpha$, where Y_{it}^a is adjusted real GDP per worker from (6), and K_{it} is the capital stock per worker. A panel for $\ln z_{it}$ is obtained as the estimated residuals from the pooled OLS regression:

$$\ln\left(Y_{it}^a K_{it}^{-\alpha}\right) = \ln X_{z1} + (1-\alpha)(t-1)\ln\gamma_z + \ln z_{it}.$$

Implicitly, $\{A_{zit}\}_{t=1}^T$ is trend-stationary with a trend component common to all countries. The capital share α is equal to 1/3 and constant across countries and time.

The (gross) national saving rate is computed residually as $s_{it} = 1 - c_{it} - g_{it}$, where c_{it} and g_{it} are, respectively, the nominal private consumption and the nominal public spending shares of nominal GDP, in current international prices (variables CC and CG in the PWT). This procedure was also followed, for instance, in Backus et al. (1992). A more accurate measure of saving would require more complete data on real GNP than that available in the PWT. A further potential issue is that CC and CG are not ratios of variables measured in domestic currency. While for some purposes it might be preferable to concentrate on saving rates in current domestic currency, as indicated by Summers and Heston (1991), here I choose instead to replicate this measurement convention in the model - see footnote 13. Finally, the trade balance to GDP ratio was computed as $tb_{it} = s_{it} - i_{it}$, where i_{it} is the variable CI in the PWT.

B Numerical Algorithm

The model is solved using value function iteration and continuous-variable approximation methods. The algorithm was programmed in Fortran 90 and the computer code is available upon request. See Judd (1998) for detailed descriptions of the numerical algorithms referenced in this appendix. The main step finds the real world interest rate that clears the market for foreign bonds, using Brent's algorithm. I first solve an individual country's problem given r. Given r, the Bellman equation is solved on a finite number of states. The outcome is $v(\omega, z, \theta; r)$, $b'(\omega, z, \theta; r)$ and $k'(\omega, z, \theta; r)$ at those states. I then use the decision rules to evaluate the market outcome. I next comment on the most important elements of the algorithm. To select two interest rates that bracket the world demand for foreign assets I use the fact that the following property holds

$$\lim_{r \to \gamma^{\sigma}/\beta - 1} \int_S b'(\omega, z, \theta; r)\, d\psi = +\infty,$$

and pick a very low interest rate (eventually negative) for the lower bound. Note that $\gamma^{\sigma}/\beta - 1$ is the equilibrium interest rate under complete markets. Since $\underline{b}(\omega, z, \theta) \equiv -(\eta/\gamma)\,\omega \leq 0$, the continuity of the world demand for foreign bonds ensures the existence of the equilibrium. See Clarida (1990) for a formal proof of these claims in a related setup.

In discretizing the state-space I choose a much finer grid for lower values of the state variables, where the value function exhibits significant curvature. The range of the discretized S is chosen so that it contains more than 99.99% of the country states in equilibrium. The number of grid points is $N_{\omega} = 79$, $N_z = 44$ and $N_{\theta} = 14$.

I solve the following discretized version of the Bellman equation, with p-point Gaussian quadrature (in practice $p = 4$) to approximate the conditional expectation. The Gaussian quadrature weights are denoted by $\{\lambda_i\}_{i=1}^{p}$ and the corresponding abscissas by $\{x_i\}_{i=1}^{p}$.

$$\begin{aligned} v^n(\omega_f, z_g, \theta_h; r) \quad &= \quad \max_{c, b', k'} \left\{ u(c) + \beta \gamma^{1-\sigma} \frac{1}{\pi} \sum_{i=1}^{p} \sum_{j=1}^{p} \lambda_i \lambda_j v^{n-1}\left(\tilde{\omega}_{ij}, \tilde{z}_i, \tilde{\theta}_{ij}; r\right) \right\} \\ & \quad \text{s.t.} \quad \text{constraints} \end{aligned}$$

for $n = 1, 2, \ldots$, and for all $(\omega_f, z_g, \theta_h)$ in the grid. The arguments $\tilde{\omega}_{ij}$, $\tilde{z}_i$ and $\tilde{\theta}_{ij}$ are

$$\begin{aligned} \tilde{\omega}_{ij} &= \tilde{z}_i \left(k'\right)^{\alpha} + (1 - \delta)\, \tilde{\theta}_{ij} k' + (1 + r)\, b' \\ \tilde{z}_i &= z_g^{\rho_{zz}} \theta_h^{\rho_{z\theta}} \exp\left\{\mu_z + \sqrt{2}\sigma_z x_i\right\} \\ \tilde{\theta}_{ij} &= z_g^{\rho_{\theta z}} \theta_h^{\rho_{\theta\theta}} \exp\left\{\mu_{\theta} + \sqrt{2}\left[\frac{\sigma_{z\theta}}{\sigma_z} x_i + \sqrt{\sigma_{\theta}^2 - \left(\frac{\sigma_{z\theta}}{\sigma_z}\right)^2} x_j\right]\right\}. \end{aligned}$$

These expressions obtain after performing the multivariate change of variables required to apply the appropriate Gaussian quadrature formula (Gauss-Hermite, with a product rule to compute the double integral). The two main stages of the maximization step are described next, together with the procedure used to compute the market outcome.

Optimization

The maximization step is carried out using a combination of the Newton-Raphson, downhill simplex and Brent algorithms (the last one with explicit computation of the gradient). I first use the Newton-Raphson method to solve the unconstrained maximization problem when choosing both assets. I then check whether the borrowing constraint was violated: if it was not, then the solution was obtained; if it was violated, I set $b' = -(\eta/\gamma)\,\omega$ and use Brent's method to find the maximum in the k dimension. On the early iterations, I use the downhill simplex algorithm instead of Newton-Raphson's, since the first one is more robust to bad initial guesses. Large efficiency gains can be obtained by recognizing that $b'(\omega, z, \theta; r)$ is monotonic in all arguments and by implementing policy function iteration.

Interpolation

Since $\tilde{\omega}_{ij}$ will in general not belong to the grid, I use cubic spline interpolation to evaluate v^{n-1} at points outside the grid. When the relevant points are outside the bounds of the grid space (which happens a very small number of times) I use the cubic spline to extrapolate. In the numerical experiments, this seemed to be reasonably accurate.

Given that a substantial degree of curvature is also present in the remaining dimensions of S, cubic spline interpolation is appropriate to compute the value function associated with $\tilde{z}_i$ and $\tilde{\theta}_{ij}$, which are in general outside the grid as well. In all cases, I pin down the endpoints of the cubic spline by setting the first derivative at those endpoints equal to the first derivative of a polynomial of degree m through the endpoint and the m adjacent points (a variant of the secant Hermite spline). The degree m is set so that the curvature at the endpoints is appropriately captured by the cubic spline. I first interpolate on the θ and z directions and finally on the ω direction. This particular sequence is dictated by efficiency considerations, since only $\tilde{\omega}_{ij}$ depends on decisions. Therefore, only one evaluation of the cubic spline needs to be carried out for each evaluation of the right-hand-side of the Bellman equation.

Market Outcome

To compute the solution for each r, I require that the maximum absolute percent difference between two consecutive iterations, in terms of decision rules, has to be lower than 10^{-6}. Once the decision rules are found, I compute decision rules on a much finer grid, with 330 points for wealth, 135 points for z, and 105 points for θ. To compute the distribution of countries over S, $\psi(r)$, I use the system's transition function to simulate a long (size $T = 2$ million) time-series for an individual country. A sample size this large is important for accuracy since some of the endogenous variables are very persistent. I use a simple three-dimensional linear interpolation scheme to update the decision rules. Linear interpolation with a large number of grid points is a suitable procedure to deal with the kinks exhibited by the decision rules arising due to the presence of the borrowing constraint. In order to preserve accuracy, I discard innovations leading to pairs (z, θ) outside the discretized S (these innovations corresponding to less than 0.01% of the total). Observations for ω outside the grid space (less than 0.01% of the total) are found by linear extrapolation. In practice, this amounts to concentrating on a compact state-space.

The world excess demand for net foreign assets, $\varphi(r) \equiv \int_S b'(\omega, z, \theta; r)\, d\psi(r)$, is approximated by $\tilde{\varphi}(r) = 1/T \sum_{t=1}^{T} b_t$, where $\{b_t\}$ is a sample sequence of net foreign asset holdings. The algorithm terminates when the relative difference between the current guess for r and the solution is lower than 10^{-4}.